ESTIMATION OF YOUNG'S MODULUS AND FAILURE STRESSES IN THE HEN'S EGG SHELL

by

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INTRODUCTION

The number of cracked eggs received by grading stations has doubled in the past fifteen years. A current estimate of the annual loss to Canadian egg producers is four million dollars (10). The growing problem of cracked eggs is aggravated by the trend toward higher egg production per hen and more extensive use of egg-handling equipment. The successful design of egg-handling equipment requires reliable data on physical and mechanical properties of eggs.

Rehkugler (5) measured Young's modulus and failure stresses of ten egg shells using ring or semicircular portions of shell. The results showed considerable variation and the author acknowledged the need for further study of egg shell mechanical properties. Sluka *et al.* (8) and Hammerle and Mohsenin (3) subjected shells to internal hydrostatic pressure and calculated stresses in the shell at failure. Voisey and Hunt (11) used Rehkugler's values for Young's modulus and ultimate strength along with shell theory to predict force at failure of shells loaded between flat plates but found their estimates to be considerably lower than the measured forces at failure. An alternate approach to this problem is the estimation of egg shell elastic properties from deformation behavior. This paper describes the use of shell theory, shell geometry and deformation characteristics to estimate Young's modulus and failure stresses of egg shells.

THEORY

Analysis of stresses in the egg shell was simplified for this work by assuming the egg to be spherical so that existing solutions could be used. Reissner (6) provided solutions for a spherical shell loaded at a point and for a spherical shell with load uniformly distributed over a small circular area. For a point load, total deformation of the shell is

$$d_t = \frac{PR\sqrt{3(1-V^2)}}{2\ ET^2} \quad \text{........1}$$

where d_t = total deformation in the direction of applied force

P = applied force

R = radius of curvature of shell

V = Poisson's ratio of shell material

E = Young's modulus of shell material

T = shell thickness

If deformation, load, radius of curvature and shell thickness are measured, and a value for Poisson's ratio assumed, Young's modulus may be determined using

$$E = \frac{PR\sqrt{3(1-V^2)}}{2\ d_t\ T} \quad \text{........2}$$

Stresses in meridial and latitudinal directions (membrane stresses) at the axis of load may be calculated from

$$S_m = \frac{-P\sqrt{3(1-V^2)}}{8\ T^2} \quad \text{..........3}$$

Bending stresses may also be computed for positions in the shell adjacent to the loading axis; however, they become infinite at the apex.

For a load uniformly distributed over a small circular area, deformation of the shell is

$$d_s = \frac{4\ PR\sqrt{3(1-V^2)}}{\Pi\ ET^2}\left[\frac{\ker' u}{u} + \frac{1}{u^2}\right] \quad \text{..........4}$$

where $u = \dfrac{r\sqrt[4]{12(1-V^2)}}{\sqrt{RT}}$ 5

d_s = shell deformation in the direction of applied force

r = radius of the circular loading area

ker' = is a Kelvin-type Bessel function (1, 2)

Young's modulus is then

$$E = \frac{4\ PR\sqrt{3(1-V^2)}}{\Pi\ d_s\ T^2}\left[\frac{\ker' u}{u} + \frac{1}{u^2}\right] \quad \text{..........6}$$

Membrane stresses at the centre of the loading area are

$$S_m = \frac{-P\sqrt{3(1-V^2)}}{\Pi\ T^2}\left[\frac{\ker' u}{u} + \frac{1}{u^2}\right] \quad \text{..........7}$$

and bending stresses are

$$S_b = \pm\frac{3P(1+V)}{\Pi\ T^2}\left[\frac{\mathrm{kei}' u}{u}\right] \quad \text{......................8}$$

where kei' is a Kelvin-type Bessel function.

Timoshenko and Goodier (9) provide a method for calculating contact area between a sphere and flat plate. The radius of the contact area is

$$r = \left[0.75\ P\ R\ (k_p + k_s)\right]^{1/3} \quad \text{......................9}$$

where $k_p = \dfrac{1 - V_p^2}{E_p}$ 10

$$k_s = \frac{1 - V_s^2}{E_s} \quad \text{....................11}$$

V_p = Poisson's ratio of plate material

V_s = Poisson's ratio of sphere material

E_p = Young's modulus of plate material

E_s = Young's modulus of sphere material

Deformation in the direction of applied force, due to local flattening at the two loading surfaces is

$$d_f = \left[\frac{9 \, P^2 \, (k_p + k_s)^2}{2 \, R} \right]^{1/3} \quad \ldots 12$$

The total deformation is due to local flattening at the loading surfaces and bending of the shell, that is,

$$d_t = d_f + d_s \quad \ldots\ldots\ldots\ldots 13$$

Estimates for Young's modulus of egg shell may be used in equation 11 and compared with calculated values from equation 6 in a trial-and-error solution. Equation 9 must be used with reservation for approximation of the loading area. Unlike a solid sphere, a shell experiences slight bending that increases its radius of curvature, and also there is a parabolic load distribution at the contact surface rather than a uniformly distributed load as is assumed in shell theory.

PROCEDURE

A total of 299 eggs were collected from fifty Single-Comb White Leghorn pullets during August and September, 1967. An attempt was made to obtain equal numbers of eggs from all birds. A standard laying ration was fed and individual cages were used to allow identification of eggs produced by each hen. Egg diameter at the equator was measured before each egg was compressed at 44 ± 2 microns per second parallel to its major axis between flat plates in the apparatus (figure 1) described by Richards and Staley (7). The egg was held in an upright position with a pliable foam rubber support while force and deformation were measured with an X-Y recorder. Egg contents were discarded and shell membranes removed in boiling aqueous sodium hydroxide. The shells were rinsed in water, dried and the average thickness at the egg equator was measured using a dial micrometer fitted with a spherical anvil.

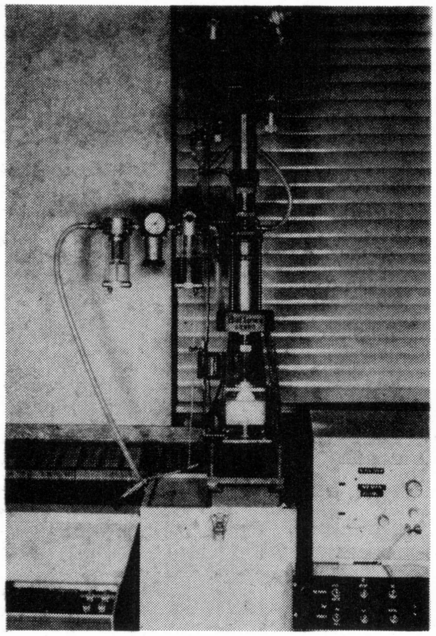

Figure 1. Compression testing machine.

Half the diameter of the egg at the equator was assumed to be the radius of shell curvature. Young's modulus and Poisson's ratio for the steel loading plates were 21,100 kg/mm² and 0.25 respectively. Poisson's ratio for the egg shells was estimated as 0.25. It should be noted that varying the estimate for Poisson's ratio by ± 0.05 alters the calculated value of Young's modulus by about 1.5 percent.

A computer programme was written to calculate Young's modulus and membrane stresses for a point load (equations 2 and 3) and Young's modulus, membrane stresses and bending stresses for uniformly distributed load (equations 6 to 8). Polynomial approximations were used in place of the Bessel functions (1). Means and standard deviations were calculated for the pooled data and analyses of variance were used to test whether eggs from different birds possessed different mechanical properties.

RESULTS AND DISCUSSION

Estimates for Young's modulus of individual egg shells, using point load and distributed load assumptions, differed by less than three percent. Because of the wide variation in the estimates of this modulus between eggs, the simpler point load calculations should be satisfactory for most purposes. On the basis of assumptions made and precision of measurement, error limits on these calculations are estimated at ± 25 percent. That is, if egg shells satisfy the conditions imposed by the theories leading to equations 2 and 6, Young's modulus for an egg shell is within 25 percent of the calculated value.

The values obtained from Young's modulus (table I) are about three times greater than the average value of 1520 kg/mm² reported by Rehkugler (5). They are, however, only slightly lower than the average published values (4) for limestone (5900 kg/mm²) and marble (5600 kg/mm²) which are similar in chemical composition to egg shells. A lower Young's modulus for egg shells, than for limestone or marble, may be due to the presence of numerous pores and an organic matrix in part of the shell. The range of Young's modulus

TABLE I. MEANS AND STANDARD DEVIATIONS

	Mean	S.D.
Force at failure, kg	3.186	0.778
Deformation at failure, mm	0.131	0.025
Radius of curvature, mm	21.76	0.655
Shell thickness, mm	0.314	0.028
Young's modulus (point load), kg/mm²	4571.0	909.4
Membrane stresses "	-6.758	1.329
Young's modulus (distributed load), kg/mm²	4694.0	939.7
Membrane stresses "	-6.665	1.307
Bending stresses "	±46.66	8.699

TABLE II. F - RATIOS FROM ANALYSES OF VARIANCE TO TEST DIFFERENCES AMONG BIRDS AND DIFFERENCES AMONG EGGS OF INDIVIDUAL BIRDS

	Among Birds [1]		Among Eggs [2]	
Force at failure	8.08	**	5.44	**
Deformation at failure	2.90	**	5.00	**
Radius of curvature	36.14	**	6.91	**
Shell thickness	14.24	**	15.46	**
Young's modulus (point load)	4.62	**	7.28	**
Membrane stresses "	6.58	**	0.32	N.S.
Young's modulus (distributed load)	4.44	**	7.47	**
Membrane stresses "	6.59	**	0.33	N.S.
Bending stresses "	6.84	**	0.64	N.S.

1 Degrees of freedom, 49/244

2 Degrees of freedom, 5/244

** $P \leqslant 0.01$

for 95 percent of the shells tested lies within 2800 to 6500 kg/mm².

Membrane stresses in meridial and latitudinal directions were considerably lower than bending stresses near the axis of load when the shells were crushed between flat plates. Since the greatest stresses in the shell were due to bending, failure may well have occurred when the material at the inner shell surface reached its ultimate tensile stress as was suggested by Voisey and Hunt (11). The values of maximum stresses in the shell may not be valid estimates of the ultimate tensile strength of shell material because of interacting bending and membrane stresses. Nevertheless, failure stresses were found to be considerably higher than those reported by Rehkugler (5), Sluka *et al.* (8) and Hammerle and Mohsenin (3).

Analysis of variance (table II) showed highly significant ($P \leqslant 0.01$) differences among birds and among successive eggs from individual birds for Young's modulus. Stresses at failure were highly significantly different when the eggs were considered on a bird-to-bird basis; however, there was no significant difference among successive eggs from the same hen.

SUMMARY

Shell theory was used with physical properties and crushing strength of 299 eggs to estimate Young's modulus and failure stresses of egg shells. Average values for Young's modulus were 4571 and 4694 kg/mm² when assuming a point load and a load uniformly distributed over a small area respectively within estimated limits of ± 25 percent. Membrane stresses at failure for point loads averaged –6.76 kg/mm². For loads distributed over a small area average membrane stresses were –6.67 kg/mm² and bending stresses were ± 46.66 kg/mm² at the shell apex.

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