COMPUTER ANALYSIS OF FORCES IN GRAIN BINS WITH MULTIPLE TIE RODS

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INTRODUCTION

The major proportion of grain harvested each year on the Canadian prairies must be stored on the farm, for a few months at least. During recent years there has been a trend toward the use of circular bins of steel and plywood, but a significant quantity still is stored in rectangular bins of wood-frame construction. Tie rods are normally used in bins of this type to relieve the load on the studs. The size and spacing of tie rods used are highly variable and frequently bear little relation to good engineering practice. However, it is not a simple matter to establish the optimum spacing of tie rods, particularly for larger bins where two or more rows of ties are used. First, the structure is statically indeterminate and second, the pressure against the bin wall is not predictable to a high degree of accuracy.

The purpose of the study reported here was to develop a method of analyzing as an indeterminate structure, a grain bin wall supported by one or more tie rods. Design data required from this analysis would include reactions at the top and bottom of the wall and at the tie rods, along with the magnitude and distribution of shear forces and bending moments in the wall. This could be done longhand for a particular bin with specific tie rod locations but to do so for a variety of tie locations to optimize the design becomes extremely tedious. If the effect of parameters, such as bin shape and angle of repose, on shear and moment distribution are added, it is obvious that the services of a computer are required.

Numerous studies have been made of pressures on grain bin walls with some variation in results reported. The method of analysis was therefore made flexible so that any magnitude or distribution of grain pressure could be inserted with minimal effort.

RECEIVED FOR PUBLICATION JUNE 22, 1972

REVIEW OF LITERATURE

Much of the work on grain pressures has been on deep bins, a deep bin being defined as one in which the height to width ratio is such that a plane of rupture from the base of one wall intersects the opposite wall rather than the grain surface. The Janssen formula is commonly used to calculate pressures in such bins since it takes the vertical load on the walls into account and predicts lateral pressures that compare favorably with measured values. Various investigators have noted variations in lateral pressures due to fluctuations in temperature and moisture content and due to unloading from one side of the bin.

For shallow bins, the Coulomb equation or special uses of it, such as the Rankine equation, are frequently used. Both Saul[a] and Stewart (6) noted that the flexibility of the walls have a considerable effect on measured pressure. Presumably friction forces in the grain mass reduce pressures against a wall that moves away from the grain and increase the pressures if the wall moves inwards. Where a very wide bin is heaped with grain, the surcharged fill can produce significantly greater pressures.

Although many references to pressures in grain bins were found, relatively few articles on the design of bin walls were noted. Of these, most were concerned with the design of large storage bins made of steel or reinforced concrete. Stahl (5) developed a series of nomographs that gives shears and bending moments in studs, as well as reactions for bin walls with no tie rods or with one row of tie rods. Backus[b] attempted to further Stahl's work by presenting similar data for bins with two sets of tie rods and produced numerous charts that tended to confuse rather than simplify the problem. The work of Backus was based on an extension of the three moment equation that can be found in most texts on strength of materials. Both Stahl and Backus used Janssen's formula for grain pressure.

[a] Saul, R.A. 1959. Effect of bin design factors on pressures of shelled corn. Paper No. 59-820, Amer. Soc. Agr. Eng., St. Joseph, Michigan.

[b] Backus, D.A. 1964. Use of digital computers in solving problems in statically indeterminate structures: two-tie grain bins. ASAE Paper presented at Oct. Meet. Amer. Soc. Agr. Eng., Vancouver, British Columbia.

THEORY AND PROGRAM DEVELOPMENT

It was decided that analysis of the bin wall as a multispan continuous beam using a finite element method would be a very suitable procedure for this study. The finite element method of analysis of structures is no longer a new technique; therefore, the description of this method will be kept to a minimum. A computer-oriented analysis of a continuous beam can be found in many of the numerous texts now available on finite elements and structural analysis (2, 3, 4).

The bin wall may be idealized as shown in Figure 1, where P1, P4, P2, and P3 represent the reactions of the top and bottom of the bin wall and the reactions of the tie rods, respectively. The wall load due to the pressure of the grain is shown as a distributed load.

If we consider the element of this beam to be one span, then the stiffness matrix of the element can be established as given in Figure 2. The following assumptions are made: (i) no external moments are introduced to the beam at

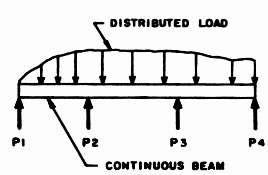

Figure 1. Bin wall idealization.

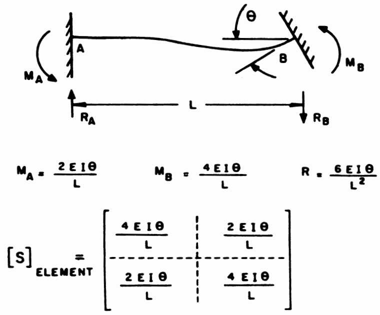

Figure 2. Element stiffness matrix.

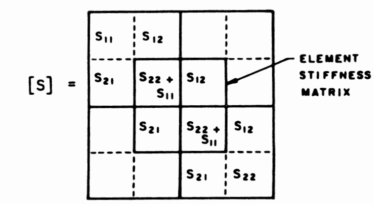

Figure 3. Structure stiffness matrix.

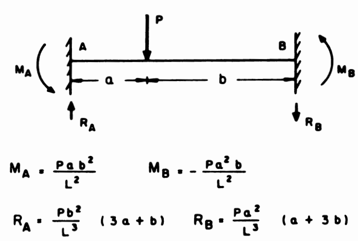

Figure 4. Fixed-end action due to load.

any point; (ii) only rotational displacements occur at the structure nodes (tie rod locations); (iii) displacements are small enough for the laws of linear superposition to hold and the elementary beam theory may be used. The matrix elements or stiffness coefficients are based on fixed-end actions caused by end displacements when these displacements are set equal to unity (2). The stiffness matrix for the whole structure is constructed by simply placing these element stiffness matrices on a diagonal with an overlapping arrangement where different elements have common nodes (Figure 3).

The finite element method requires that loads be applied only at nodes, and that distributed member loads must be replaced by equivalent concentrated nodal loads. This can be done by discretizing the distributed load and evaluating the fixed-end actions caused by each discrete load and then, due to the law of linear superposition, adding these fixed-end actions to find the equivalent nodal loads of the structure. The fixed-end actions caused by a concentrated discrete load are given in Figure 4.

In matrix notation, this would be:

$$[A_D] = [S][D] \quad \ldots\ldots \quad (1)$$

where $[A_D]$ is the column matrix of nodal actions due to the nodal displacements, $[S]$ is the square structure stiffness matrix, and $[D]$ is the nodal displacement column matrix. By inversion of $[S]$, equation (1) can be changed to:

$$[D] = [S]^{-1}[A_D] \quad \ldots\ldots \quad (2)$$

Knowing $[S]$ and $[A_D]$, $[D]$ can be established by means of equation (2). Now the nodal reactions can be found by:

$$[A_R] = [A_{RL}] + [A_{RD}][D] \quad \ldots \quad (3)$$

where $[A_R]$ is the column matrix of nodal reactions, $[A_{RL}]$ is the column matrix of nodal loads not corresponding to nodal displacements, and $[A_{RD}]$ is the rectangular matrix of nodal loads corresponding to unit displacements at the node. The matrix $[A_{RL}]$ can be found using Figure 4, and $[A_{RD}]$ can be found using Figure 2. Once the moments and reactions at the nodes are known by means of $[A_R]$, the problem is essentially solved and it becomes a simple matter to evaluate the moment and shear distribution throughout the beam. This is normally sufficient for the structural design of the beam.

RESULTS AND DISCUSSION

Program Accuracy Check

The program was tested by analyzing a uniformly loaded continuous beam with four points of support. The results of the program agreed very well with the values of the reactions, maximum shear, and maximum moment as given in a structural design handbook (1).

Wall Forces Based on Janssen's Formula

Janssen's formula is given as:

$$L = \frac{wR}{\mu}\left(1 - \exp\frac{(-K\mu H)}{R}\right) \quad \ldots \quad (4)$$

where L is lateral pressure in lb/ft^2, w is density of grain in lb/ft^3, R is the hydraulic radius of bin, K is the ratio of lateral to vertical pressure, and μ is the coefficient of friction of grain on the wall. This equation was used as the distributed load input for the beam analysis program.

Moment and shear distributions for several tie locations were obtained. Examples of the effects of tie spacing on the shear and moment distributions are shown in Figures 5 and 6. It is evident that the location of the top tie has a substantial effect on bin design. The top tie location, denoted by T, and bottom tie location, denoted by B, are ratios of tie depth, measured from the top of the wall, to total wall height. Values of other parameters were $K = 0.5$, $\mu = 0.4$, and $w = 50$ lb/ft^3.

A more complete series of program runs was made, changing various parameters, to determine the effects of tie location, bin size, and bin shape on the shear and moment distributions in the bin wall. The information needed for the design of a bin wall includes the reactions at the top and bottom of the wall as well as the reactions at the tie rods, and the maximum bending moment and the maximum shear force in the wall. It should be noted that the maximum shear force is less than the maximum tie rod reactions, for the reaction force includes the shear force just above and just below the tie. A large amount of data for the various combinations was obtained from the

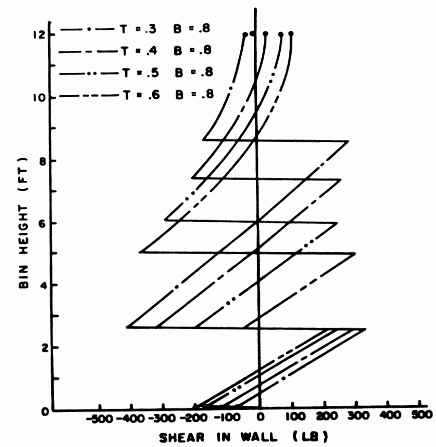

Figure 5. Effects of tie location on shear distribution.

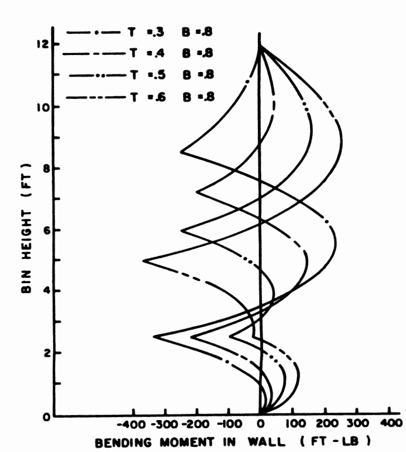

Figure 6. Effects of tie location on moment distribution.

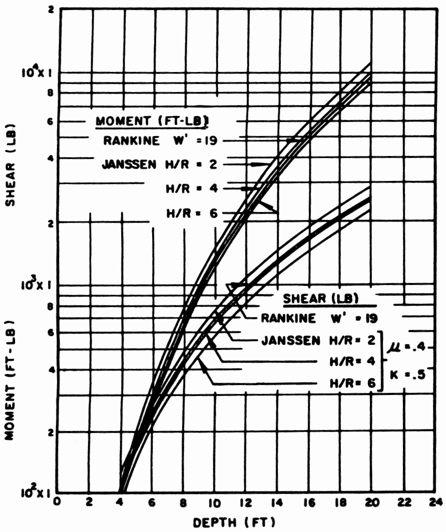

Figure 7. Effects of bin diameter on maximum shear and moment in bin with no ties.

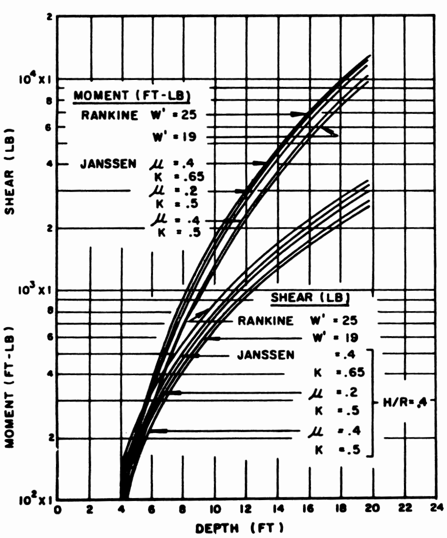

Figure 8. Effects of various parameters on maximum shear and moment in bin with no ties.

computer in a fairly short time. The next problem was to state the results in a reasonably simple and useable form.

Figures 7 and 8 show maximum shear and maximum moment values for bins with no tie rods. Values given are for studs spaced 1 ft (0.3 m) on center. Calculations were made for bin heights of 8, 12, 16, and 20 ft (2.4, 3.7, 4.9, and 6.1 m). For each height, bin diameters were chosen to provide height to hydraulic radius ratios of 2, 4, and 6. Figure 7 illustrates the variation in shear and moment produced by these values of height to hydraulic radius ratios. Figure 8 shows the effects of changing the values of K and μ. Although the effects seem relatively small, the logarithmic scale used for the ordinate is rather deceiving. There is sufficient difference to be worth considering. Additional curves, similar to those in Figures 7 and 8, could be produced for other combinations of the variables, but for most practical situations, reasonably accurate values can be estimated from these figures.

Table I shows the effects that the locations of two tie rods have on the tie rod reactions, shears, and moments. In this table, P2, P3, and P4 are reactions at the tie rods and the bottom of the wall (Figure 1). Results are tabulated as ratios of the forces or moments without tie rods to the forces or moments with tie rods for various tie rod positions. Consequently, a large value of the ratio indicates a small force or moment in the bin with tie rods. Thus, a high percentage change in a large ratio does not necessarily mean a great change in the absolute value of the force or moment. The values in Table I apply to the whole range of values of H/R, μ, and K with reasonable accuracy. A change in the shape of the pressure distribution curve changed these ratios slightly but the maximum deviation from the mid-range value was less than 10% except where noted by a superscript and corresponding footnote.

A negative value in any reaction ratio implies that that particular reaction is in compression. That this can happen in theory can be envisioned by supposing that P1 and P2 are close together at one end of the wall, and P3 and P4 are close together at the other end. When a load is applied to the middle of the wall, the middle would deflect outwards whereas both ends would tend to deflect inwards. However, an initial assumption or boundary condition was that structure nodes (reaction locations) could have no linear displacement. Therefore, in this instance, P1 and P4 would have to be opposite in sign to P2 and P3.

The ratio of V (no tie) to P1, the top reaction, did not behave in a manner similar to the rest of the data in Table I; however, it was noted that the value of P1 itself changed very little with bin diameter for a given height. The average value of P1 for a particular height and tie spacing for all diameters considered is given in Table II. The maximum deviation of any value from the average was less than 10%.

Wall Forces Based on Rankine's Formula

Rankine's formula for a bin with level fill is given as:

$$P = \frac{wh\,(1 - \sin\phi)}{(1 + \sin\phi)} \quad . \; . \; . \; . \; . \quad (5)$$

where w is the density of the grain, h is the grain depth, and ϕ is the internal angle of friction of the grain. Unlike Janssen's formula, Rankine's is a straight-line formula and can be considered as an equivalent fluid pressure formula by:

$$P = w'h \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

where

$$w' = \frac{w(1 - \sin\phi)}{(1 + \sin\phi)} \quad . \; . \; . \; . \; . \quad (7)$$

is the equivalent fluid density.

TABLE I COMPARISON OF REACTIONS, SHEARS, AND MOMENTS WITH AND WITHOUT TIES BASED ON JANSSEN'S FORMULA

T	.3	.3	.3	.3	.4	.4	.4	.5	.5
B	.5	.6	.7	.8	.6	.7	.8	.7	.8
V (no tie)/P2	33.8§	4.71†	2.53	1.85	5.00‡	2.39	1.80	1.92	1.54
V (no tie)/P3	.975	1.14	1.18	1.07	1.20	1.35	1.28	1.72	1.73
V (no tie)/P4	1.97	2.38	3.34	10.1‡	2.38	3.12	6.32†	3.05	4.60
V (no tie)/V	1.63	1.88	2.23	1.98	1.89	2.34	2.41	2.42	2.99
M (no tie)/M	3.96	5.93	7.37	5.86	5.98	9.39	8.69	9.23	8.44

† Deviation of range extremes from midpoint < 15%.
‡ Deviation of range extremes from midpoint < 30%.
§ Deviation of range extremes from midpoint < 60%.

TABLE II VALUES OF TOP REACTION (P1) (lb) BASED ON JANSSEN'S FORMULA

T	.3	.3	.3	.3	.4	.4	.4	.5	.5
B	.5	.6	.7	.8	.6	.7	.8	.7	.8
H = 8	26.8	16.1	1.64	−14	29.2	22.9	14.3	43.6	41.5
12	61.3	42.7	9.89	−28	72.0	60.2	41.7	97.8	92.4
16	110.	74.1	23.0	−46	124.	103.	67.0	174.	164.
20	173.	114.	29.3	−70	199.	166.	118.	271.	255.

TABLE III COMPARISON OF REACTIONS, SHEARS, AND MOMENTS, WITH AND WITHOUT TIES BASED ON RANKINE'S FORMULA

T B	.3 .5	.3 .6	.3 .7	.3 .8	.4 .6	.4 .7	.4 .8	.5 .7	.5 .8
V (no tie)/P1	16.6	26.4	**.*‡	−29.2	15.7	20.1	34.6†	11.5	12.0
V (no tie)/P2	−11.2	6.59†	2.94	2.07	6.73†	2.77	2.01	2.23	1.72
V (no tie)/P3	.992	1.16	1.20	1.08	1.18	1.34	1.26	1.65	1.66
V (no tie)/P4	1.92	2.25	3.08	7.75†	2.26	2.92	5.60	2.82	4.30
V (no tie)/V	1.66	1.88	2.20	2.04	1.88	2.29	2.44	2.36	3.05
M (no tie)/M	3.75	5.52	7.10	5.76	5.50	8.46	8.26	8.82	9.15

† Deviation of range extremes from midpoint $<15\%$.
‡ Ratios >100, maximum P1 <20 lb.

For ϕ of 27° and $w = 50$ lb/ft^3, w' is approximately 19 lb/ft^3. Rankine's formula, with $w' = 19$, was used as the wall loading for the same bin heights and tie spacings as was Janssen's. The variation of bin diameter was excluded since Rankine's formula is independent of this parameter.

The results of these calculations are given in Table III. In this instance, V (no tie) and M (no tie) were determined algebraically as $(w'h^2)/3$ and $(w'h^3)/(9\sqrt{3})$, respectively. In Table III, as in Table I, the values given represent the midpoint of the range of ratios obtained for a particular tie spacing. Where any value deviated more than 10% from this midrange value, it is denoted in the table by a superscript and explained in a footnote to the table.

For bins with heaped fill or surcharge, a larger value of equivalent fluid density might be used. Figure 8 shows curves using a value of 25, which could be applicable to surcharged fill in a narrow bin or to a level fill with grain having an internal angle of friction of 19°. For a surcharged fill in a very wide bin a value of 32 for w' would be more appropriate.

Sample Calculation for Bin Design

If it were decided to design the wall for a bin 12 X 12 X 12 ft (3.7 X 3.7 X 3.7 m) based on Janssen's formula, Tables I and II would provide the ratios of V (no tie)/V and M (no tie)/M for this bin. For a number of cases tried, it was found that if wood was the construction material, shear was the limiting criterion for design. In this instance, it would seem that T = 0.5 and B = 0.8 might be a good choice since V is relatively small. If these values of T and B are assumed, and using K = 0.5 and $\mu = 0.4$, then from Figures 7 or 8 the values of moment and shear with no tie rods are 2150 and 920, respectively. Using the coefficients from Tables I and II the values of P1, P2, P3, and P4 are 92.4, 596, 532, and 200 lb (41.9, 270.6, 241.5, and 90.8 kg), respectively, for studs spaced one foot (0.3 m) on center. Similarly, the maximum shear in the stud is 308 lb (139.8 kg) and the maximum moment is 25 ft-lb (35.2 m-kg). Having this data, the designer can proceed to select size and spacing of studs and tie rods, size of wales, and to design appropriate connections.

Program Operation

This program was written in Basic language and run on a Hewlett-Packard time-share system with a small storage. The program was developed upon basic principles of structural analysis and could be, if required, easily extended to any number of tie rods, or used with most types of assumed wall loading.

CONCLUSIONS

The following conclusions can be drawn from this study:

1. Of the values tested, the lowest shear in the wall was provided with the top tie at 0.5 times the depth and the bottom tie at 0.8 times the depth for both Rankine's and Janssen's formula.

2. With few exceptions, the ratios of V (no tie) to P2, P3, P4, and V, for a given tie spacing, did not change with bin height or diameter in the ranges tested.

3. The ratio of M (no tie) to M, for a given tie spacing, did not change with bin height or diameter in the ranges tested.

4. The use of a computer program based on the finite element analysis of a continuous beam with four supports is a convenient way of analyzing a grain bin with two rows of tie rods.

SUMMARY

A computer program based on the finite element method of analyzing a continuous beam with four points of support was used to analyze a grain bin wall having two rows of tie rods. Janssen's formula and Rankine's equation were used as assumed lateral pressures but other types of pressure distribution can be used in this method of analysis. Results of calculations on bins up to 20 feet (6.1 meters) high with hydraulic radii up to 6 feet (1.8 meters) are presented for reactions at supports and for shear and bending moment in wall members. It was found that with few exceptions the ratio of bending moment without tie rods to bending moment with tie rods varied by less than 10 percent for given tie rod locations, regardless of the shape or size of the bin. Ratios for shears and reaction forces were also remarkably consistent. This permits the design data to be presented in a condensed form that is readily useable. Tie rods located at 0.5 and 0.8 of the depth, measured from the top of the wall, minimized shear force in the framing member.

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