SOIL COMPACTION AS A FUNCTION OF CONTACT PRESSURE AND SOIL MOISTURE CONTENT

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Equations are presented which make it possible to predict the amount of soil compaction as a function of soil moisture content and contact pressure. By means of these equations the drainage coefficient can be evaluated in economic terms and decisions made regarding applied pressures and operating times.

INTRODUCTION

The compaction of soils is basically a reduction in volume of a given mass of soil. Compaction may be expressed as a change in bulk density, void ratio or porosity. Many of the soil properties such as hydraulic conductivity and liquid and vapor diffusion are affected by compaction so that the outcome is generally a reduction of crop yields.

Gill (1956) stated that the annual loss in the United States of America due to soil compaction is $1.18 billion, which results from decreasing yields and increased expenditures for energy for field tillage. Soane (1970) states that "in the context of rising costs and shortage of labour, the mechanical and economic efficiency of the machinery is generally high and it seems probable that the trend for larger, heavier and more complex equipment will continue." Therefore, decreasing the compaction, or at least controlling it, is of significance both for the producer and for the consumers of food.

Among the many factors which affect compaction of a given soil, the pressure applied by the machinery and the ambient soil moisture content are the most important. However, the compaction behavior of different soils at given pressures and soil moisture contents varies with other parameters which are as yet quantitatively unknown.

There exist relationships between pressure and porosity for a given soil moisture content, and between porosity and soil moisture content for a given pressure.

Soehne (1958) found that:

\[ N = -A \ln P + C \]  \hspace{1cm} (1)

where \( N \) is the porosity in percent, \( P \) is the pressure, \( C \) is a constant and \( A \) is the slope of a straight line in a semi-log plot. In his paper, he presented results of measured densities for different levels of pressure and soil moisture content (SMC). SMC-density relationships are well known and are usually presented as in Figure 1.

The objectives of this paper are (1) to show that there exists a combined relationship between density, pressure and soil moisture content which can be expressed by a relatively simple exponential equation, and (2) to suggest ways to apply this relationship in practice.

POROSITY – PRESSURE – SOIL MOISTURE CONTENT EQUATIONS

It is a happy coincidence that most farm field work in humid climates is carried out when the soil moisture content is between 0.4 and 0.9 of saturation. As a result of experimental work and by analyzing published results, we have found that for the range of SMC from 0.4 to 0.9 of saturation, soil porosity can be expressed by the equation:

\[ N = A_n - B_n \ln (P_R + P) - C_n \ln \theta \]  \hspace{1cm} (2)

where \( N \) is the porosity in percent, \( P \) is the applied pressure, \( P_R \) is the residual pressure, which will be explained later, \( \theta \) is the volumetric soil moisture content in percent, and \( A_n, B_n, C_n \) are constants.

Also \( \gamma_d \), the dry bulk density, can be related to compacting pressure and soil moisture content by the equation:

\[ \gamma_d = A_d + B_d \ln (P_R + P) + C_d \ln \theta \]  \hspace{1cm} (3)

The constants \( A_n, B_n, C_n \) and \( A_d, B_d, C_d \) are accordingly related to each other, since

\[ \gamma_d = \frac{G_S (100 - N)}{100} \]  \hspace{1cm} (4)

where \( G_S \) is the specific weight of the solids and \( N \) is the porosity in percent. For most agricultural soils \( G_S = 162-168 \) pcf or 2.6-2.7 g/cm³.

The relationships for the constants are

\[ A_d = \frac{G_S}{100} (100 - A_n) \]  \hspace{1cm} (7)

\[ B_d = \frac{G_S}{100} B_n \]  \hspace{1cm} (8)

\[ C_d = \frac{G_S}{100} C_n \]  \hspace{1cm} (9)

The results have been checked by comparing the calculated values with observed results from several studies. In addition, chi-square and t tests have been made to check whether or not both the observed and the calculated values are from the same population and to check the level of significance of any differences in the populations (Table I). For each of the sources, the number of measurements of the calculated constants \( A_n, B_n, C_n, A_d, B_d, C_d \), the ranges of pressures and SMC's, and the levels of significance are indicated. (Note: In order to verify the suggested equations 2 and 3 published data have been used.) Detailed results for Soehne's (1958) data are presented.
TABLE 1  CONSTANTS FOR EQUATIONS 5 AND 6 FOR SEVERAL SOURCES OF DATA

<table>
<thead>
<tr>
<th>Source</th>
<th>No. of readings</th>
<th>Soil type</th>
<th>Constants</th>
<th>Range of (\theta (%))</th>
<th>Range of (P) (psi)</th>
<th>Level of significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(A_n)</td>
<td>Lower</td>
<td>Upper</td>
<td>Lower</td>
</tr>
<tr>
<td>Raghavan et al. 1975</td>
<td>15</td>
<td>Sandy</td>
<td>98.9</td>
<td>21.8</td>
<td>31.9</td>
<td>11.1</td>
</tr>
<tr>
<td>Barnes et al. 1971</td>
<td>8</td>
<td>Green field, sandy loam</td>
<td>76.0</td>
<td>3.8</td>
<td>9.5</td>
<td>11.8</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>Yolo loam</td>
<td>107.8</td>
<td>10.9</td>
<td>15.1</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>Yolo silty loam</td>
<td>127.0</td>
<td>8.0</td>
<td>23.2</td>
<td>0.05</td>
</tr>
<tr>
<td>Soehne (1958)</td>
<td>35</td>
<td>Heavy loamy clay</td>
<td>89.3</td>
<td>10.0</td>
<td>160.0</td>
<td>0.01</td>
</tr>
</tbody>
</table>

* The numerical data from these sources have been obtained from graphs.
* The constants are for \(P_f = 5\) psi in equation 2 (see Table III).

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The suggested equation 2 describing the relationships between porosity, pressure and SMC could be useful for determining the following: (1) soil compaction effected by applied pressure at known moisture content, (2) time and drainage influences on soil compaction, and (3) the residual compaction and pressure.

### DISCUSSION

The suggested equation 2 describing the relationships between porosity, pressure and SMC could be useful for determining the following: (1) soil compaction effected by applied pressure at known moisture content, (2) time and drainage influences on soil compaction, and (3) the residual compaction and pressure.

1. **Soil Compaction Effected by Applied Pressure at Known Moisture Content**

   Using equation 5, the porosities of a certain soil with known \(A_n\), \(B_n\) and \(C_n\) values and having different values of \(\theta\) and \(P\) can be given by \(N_1\), \(N_2\). Then,

   \[
   N_1 - N_2 = \Delta N = B_n \ln \frac{P_2}{P_1} + C_n \ln \frac{\theta_2}{\theta_1}
   \]

   where \(\Delta N\) is the compaction of the soil expressed as the reduction of the porosity. Equation 12 can be reduced to

   \[
   P_2 = e^{\Delta N/B_n} \left(\frac{\theta_2}{\theta_1}\right)^{C_n/B_n}
   \]

   by which one can evaluate the pressure ratio \(P_2/P_1\) causing a certain compaction under different soil conditions. A practical application of this equation could be for determining when to undertake tractor work in the field. In order not to exceed a certain degree of compaction \(\Delta N\), one can operate in the field with a machine of pressure \(P_1\) when the SMC is \(\theta_1\), but if one waited until the SMC decreased to \(\theta_2\), one would be able to use a heavier machine of pressure \(P_2\). For example: for \(B_n = 4.0\), \(C_n = 12.0\) and \(\theta_2 = 0.85\theta_1\), then \(P_2 = 1.63 P_1\), or a machine applying 63% more ground pressure can be used to cause the same compaction. In such conditions, one can double the pressure by waiting until the SMC decreases by 20%.

   The above consideration leads one to consider the economic value of the rate of soil moisture decrease, which is a result of both evapotranspiration and drainage.

2. **Time and Drainage Influences on Soil Compaction**

   Decreases in soil moisture content are due primarily to evapotranspiration and percolation. However, for equal evapotranspiration rates, the difference in the reduction of soil moisture content is due to percolation only. Thus, for certain conditions of weather and crop, soil moisture content decrease is due primarily to different soil drainage rates. Empirically, it has been shown by a number of authors (Baver et al. 1972; Hillier 1971) that the decrease in water content in the initially wetted zone obeys the equation:

   \[
   \frac{dN}{dt} = -R(t + Q)^{-S}
   \]

   where \(R\), \(Q\), and \(S\) are constants and \(t\) is time. The constant \(S\) may be interpreted as the drainage coefficient.

   Gardner et al. (1970) have shown in Baver et al. (1972), that when the time in...
TABLE II  THE RESIDUAL COMPACTION AND PRESSURE FOR THE DATA REPRESENTED IN FIGURE 4*†

<table>
<thead>
<tr>
<th>P</th>
<th>Applied pressure (psi)</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔNp</td>
<td>Residual compaction (%)</td>
<td>18.1</td>
<td>12.6</td>
<td>9.2</td>
<td>5.5</td>
<td>3.0</td>
<td>2.2</td>
<td>1.9</td>
<td>1.4</td>
</tr>
<tr>
<td>Pr</td>
<td>Residual pressure (psi)</td>
<td>23.71</td>
<td>28.01</td>
<td>25.14</td>
<td>22.98</td>
<td>21.93</td>
<td>21.04</td>
<td>23.71</td>
<td>21.08</td>
</tr>
</tbody>
</table>

Pr = Pr = 23.45 psi. The standard deviation σ = ±2.32 psi which is ~10% pf Pr*.

† EPr equals to ∆N/∆lnP = 7.32 for the virgin line.

‡ Deviations of 10% of Pr are caused when there are deviations of 0.5% of ΔNp which may be due to errors in drawing and/or in readings.

TABLE III  OBSERVED VS. CALCULATED POROSITIES FOR Pn = 5 PSI

<table>
<thead>
<tr>
<th>P (psi)</th>
<th>P + 5 (psi)</th>
<th>θ (%)</th>
<th>N observed†</th>
<th>N calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.25</td>
<td>12.25</td>
<td>16</td>
<td>58.7</td>
<td>57.3</td>
</tr>
<tr>
<td>14.50</td>
<td>19.50</td>
<td>12</td>
<td>58.0</td>
<td>59.0</td>
</tr>
<tr>
<td>29</td>
<td>34</td>
<td>16</td>
<td>51.0</td>
<td>49.6</td>
</tr>
<tr>
<td>58</td>
<td>63</td>
<td>10</td>
<td>52.7</td>
<td>53.5</td>
</tr>
<tr>
<td>116</td>
<td>121</td>
<td>10</td>
<td>48.0</td>
<td>48.6</td>
</tr>
<tr>
<td>232</td>
<td>237</td>
<td>10</td>
<td>42.5</td>
<td>43.5</td>
</tr>
</tbody>
</table>

† From Barnes et al. (1971), p. 195.

Figure 3. The relationship between pressure ratio and time ratio for several drainage coefficient ratios, S2/S1 (for Cn/Bn = 2.5) giving equal compaction.

Figure 4. The relationship between porosity and compacting pressure for several soil moisture contents for precompacted and virgin soils.

The above calculations might also be useful in cases where a choice between two possible applied pressures has to be taken; that is, when one has to decide whether to operate on a field with a relatively low-pressure machine but at a short time after saturation, or to wait several days to operate on the field with a high-pressure machine and to cause the same compaction. In order to carry out such calculations based on equation 18, one has to know the initial pressure, P1. For a given soil for which Bn, Cn and S are known, P1 should be known in order to evaluate P2 which is the applied pressure of a required operation. This pressure, P1, can be indicated as the
residual pressure of the soil. It expresses the precompacted conditions of the soil resulting from a certain loading history.

3. Residual Compaction and Residual Pressures

The residual compaction of a given soil is defined as the difference between the calculated porosity for virgin conditions, using equation 5, and the observed porosity for a given history of pressure and soil moisture contents.

The residual pressure, \( P_r \), in equation 2, is the pressure which, when added to the applied pressure, results in the residual compaction. Intuitively, the residual pressure, as a result, has two properties: (a) it is constant, no matter at what applied pressure it is calculated, as long as the soil moisture content is the same, and (b) its influence on the actual measured porosity decreases with the increase of the applied pressure \( P \).

Determining the virgin porosity, \( N_v \), by equation 5 and the precompacted porosity, \( N_a \), by equation 2, one can get for a certain \( \theta \), in both cases:

\[
\Delta N_f = N_v - N_a = B_n \ln \left( \frac{P_r + P}{P} \right) - C_n \ln B_n \tag{19}
\]

\[
P_r = P(e^{\Delta N_f / B_n} - 1) \tag{20}
\]

If \( P_r \) is constant, the right-hand term of equation 20 forms a hyperbolic relationship between \( P \) and \((e^{\Delta N_f / B_n} - 1)\). That is, the greater the applied pressure, \( P \), the smaller the deviation between the virgin and precompacted porosity of the given soil. In other words, if \( P_r \) is constant in equation 19, \( \Delta N_f \) decreases with the increase of \( P \).

In order to check whether or not \( P_r \) is constant, several sources of data have been analyzed. One of them will be presented here. In Figure 4, a preconsolidated sample (dashed line) under natural field conditions has been transferred for further compaction in the laboratory, while the other lines are for virgin samples. The virgin line for \( \theta = 17.3\% \), which is the SMC of the preconsolidated sample, is also shown in Figure 4. The deviations \( \Delta N_f \) for \( P = 2, 6, 10, 20, 40, 60, 80 \) and 100 psi have been calculated by equation 20. The results are given in Table II.

Other evidence of \( P_r \) being constant has been obtained by detailed analysis of the data for the Yolo silty loam studied by Barnes et al. (1971), and characterized in Table III. After recalculating the porosities with \( P_r = 5 \) psi, much better correlation has been obtained in comparison with the correlation with \( P_r \) set at zero. The calculated porosities are determined using \( A_n = 126, B_n = 7.6, C_n = 18 \) and \( P_r = 5 \) in equation 2. Assuming that for the preconsolidated condition there exists a constant residual pressure, then the pressure, \( P_r \), could be the initial value used to determine the permissible applied pressure for further operations. However, to use equation 18, the constants \( B_n, C_n \) and the drainage coefficient \( S \) must be known.

Before using equation 12 for decision making, one must determine \( P_r \). This can be done by measuring the porosity (or the dry bulk density) and the SMC under a relatively low known pressure. By comparing the observed and the calculated virgin porosities, \( P_r \) can be evaluated and \( P_2 \), which is the applied pressure, can be determined as a function of the permissible compaction and as a function of time. In other words, for a given applied pressure, \( P_2 \), one can evaluate the number of days after which he can carry out field operations with \( P_2 \) to cause a certain amount of compaction.

CONCLUSIONS

It was found that the relationship between the applied pressure, soil moisture content and porosity can be expressed by: \( N_v = A_n - B_n \ln P - C_n \ln P \) for loose virgin soils and \( N_a = A_n - B_n \ln(P_r + P) - C_n \ln P_r \) for precompacted soil, and \( \theta = (0.4 - 0.9) \) of saturation.

The apparent residual pressure which causes the residual compaction seems to be constant at a certain time and its influence on the residual compaction decreases with the increase of the applied pressure.

The suggested equations make it possible to (a) predict the amount of soil compaction as a function of SMC and the contact pressure, (b) evaluate the drainage coefficient in economical terms which are the number of work days permissible from the compaction viewpoint, and (c) make decisions where a desired applied pressure and operating time are involved.


