GRAIN BIN WALL PRESSURES: THEORETICAL AND EXPERIMENTAL

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Increases in pressure during emptying of grain bins are frequently responsible for their collapse. Several theories have been advanced to predict these pressures. This paper summarizes these theories and compares the pressures predicted by them. Experimental results show that a considerable range of pressures can occur, depending on the bin characteristics described by the equations. Pressures during filling are affected by location of the filling spout and by the use of a grain spreader. The widely used Janssen equation underestimates pressures during filling for some cases and seriously underestimates pressures during emptying for most deep bin situations. An equation proposed by Lvin is recommended for estimating emptying pressures.

INTRODUCTION

The design of bulk storage for grain and oilseeds requires a compromise between the conflicting requirements of structural strength, economical construction and convenience in handling. This is further complicated by the lack of a concise statement of the pressures storages must withstand. Frequently bin wall pressures are calculated using equations of Janssen (1895). Experimental determinations of bin wall pressures on models and actual bins have confirmed that Janssen's theory is only applicable to static or filling conditions. Pressures larger than Janssen's have been reported during emptying (Jaky 1948; Turitzin 1963; Deutsch and Schmidt 1969; Pieper 1969; Clague and Wright 1973; Stoffers 1983). Many researchers have presented theories to predict high emptying pressures, but there is considerable variation in the predictions. A critical evaluation of these theories is required to develop a suitable basis for design. The purpose of this paper is to compare various grain bin wall pressure theories with some experimental work.

THEORIES OF STATIC PRESSURES

Janssen

Janssen's theory (Janssen 1895) is not capable of predicting emptying pressures, but it is widely accepted for filling (static) pressures in deep bins. Janssen's theory became popular mainly because of its simplicity. About the same time Airy (1897) presented more complex equations to predict bin wall pressures in shallow and deep bins. The pressures predicted by these theories are roughly comparable. Both theories are derived assuming static equilibrium of the stored product.

Janssen derived the following equation to predict lateral bin wall pressures, based on static equilibrium of a slice of granular material stored in a circular bin:

$$P_h = \frac{wR}{u'}(1 - \exp(-u'Kh/R)) \qquad (1)$$

where P_h = lateral bin wall pressure (kN/m^2) at depth h; w = specific weight of stored material (kN/m^3); R = hydraulic radius of storage structure, and is defined as the ratio of the cross-sectional area of the bin to the perimeter of the bin (m); K = the ratio of lateral to vertical pressure (P_h/P_v); u' = coefficient of friction of stored material on the bin wall material; and h = depth from the surface of material (m).

The constant K is the most controversial to determine. This constant has been reported to depend on the angle of internal friction of material (ϕ) (Ketchum 1919; Jaky 1948). According to Ketchum (1919), K is defined as $(1 - \sin\phi)/(1 + \sin\phi)^{-1}$. This is the expression commonly used in soil mechanics for earth in an active state of plastic equilibrium. For an internal friction angle of 27° (typical for wheat) this results in K = 0.376. Jaky (1948) suggested that K be given the value $(1 - \sin\phi)$, which for ϕ = 27° results in K = 0.546. Both these cases assume zero wall friction. Using Mohr's circle for failure of granular material near the bin wall, some researchers (Everts et al. 1977; Moysey 1979) have shown the dependence of K on both the angle of internal friction ϕ and angle of friction of stored material on bin wall ϕ'. They derived the following formula to determine K in the active case:

$$K \text{ active} = \frac{1 - \sin\phi\cos 2e}{1 + \sin\phi\cos 2e} \qquad (2)$$

where

$$2e = \sin^{-1}\left(\frac{\sin\phi'}{\sin\phi}\right) - \phi'$$

and the following formula to determine K in passive yield case:

$$K \text{ passive} = \frac{1 + \sin\phi\cos 2B}{1 - \sin\phi\cos 2B} \qquad (3)$$

where

$$2B = \sin^{-1}\left(\frac{\sin\phi'}{\sin\phi}\right) + \phi'$$

For wheat with an internal friction angle of 27° and wall friction angle of 17° these equations result in K active = 0.41 and K passive = 1.66. The active case would occur if a bin wall was moved away from the stored grain and the passive case, if the bin wall was moved towards the grain. The above values of K active and K passive imply that passive pressures can be very much greater than active pressures, but this has not been verified in deep bins. It is conceivable that dry grain that becomes wet in storage could swell and produce passive pressures. A sudden decrease in ambient temperature can cause some increase in pressure, but not to the extent of the full passive case (Manbeck 1984). From these equations, it can also be seen that accuracy in the measurement of internal friction angle and wall friction angle is very important.

Based on experimental investigations, Ketchum (1919) reported a value of K = 0.6 for wheat. Caughey et al. (1951) also found a value of 0.6 for corn and wheat. They determined this by measuring the average wall pressure and the average floor pressure, so their value does not apply to specific locations in the bin. Pieper (1969) reported a value of 0.5 for K under filling conditions and 0.6 for emptying conditions based on studies on barley in plywood bins. Changing the value of K in Janssen's equation does not greatly affect calculated emptying pressures. Walters (1973a) suggested an equation very similar to Janssen's but using K passive. His

TABLE I. LATERAL BIN WALL PRESSURES IN DIMENSIONLESS FORM $((Ph)\cdot(wD)^{-1})$ PREDICTED BY DIFFERENT THEORIES FOR $\phi = 27°$ AND $\phi' = 17°$

Grain depth to bin diam. ratio	Janssen (1895)	Lvin (static) (1970)	Lvin (dynamic) (1970)	Jenike et al. (1972b)	Walters (1973a)
1.0	3.323	0.359	0.675	0.91	0.752
2.0	0.518	0.615	1.156	1.53	0.812
3.0	0.637	0.768	1.443	1.77	0.817
4.0	0.708	0.818	1.536	1.56	0.818
5.0	0.751	0.818	1.536	0.69	0.818

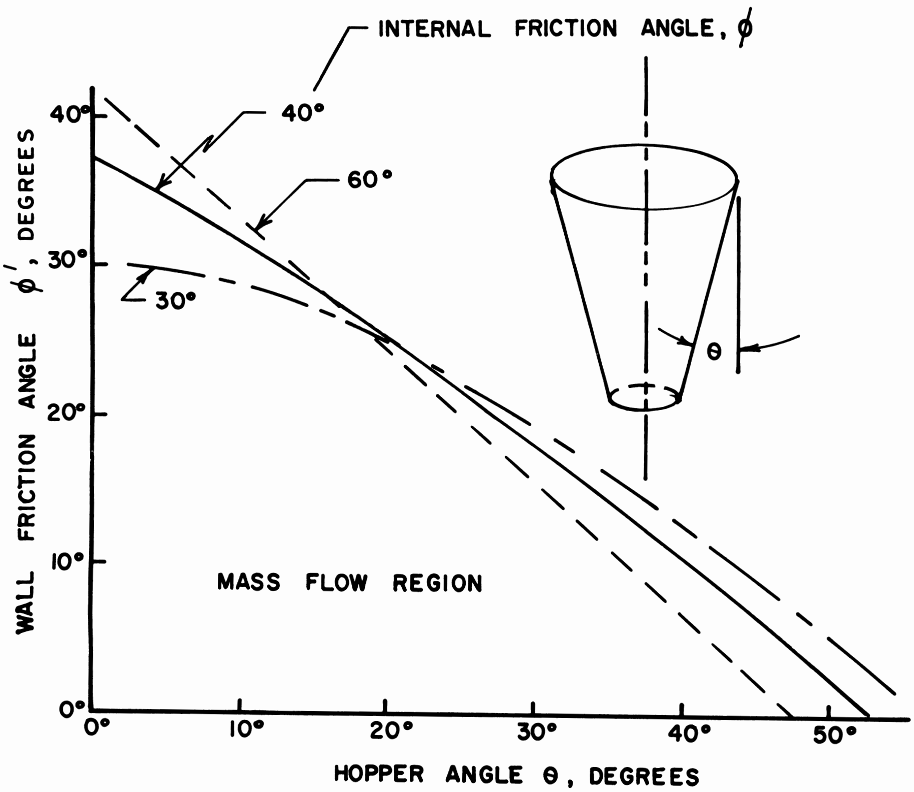

Figure 1. Limiting conditions for mass flow emptying.

theory predicts high dynamic (emptying) pressures only in the upper portions of the bin. For depth to diameter ratios, *h/D*, greater than 4.0, the emptying pressures predicted by Walter's theory are close to the filling pressures predicted by Janssen's theory (Table I).

Lvin's Theory

In an attempt to predict emptying pressures in cylindrical bins, Lvin (1970) based his derivation on the equilibrium of elementary concentric rings instead of horizontal slices. This allowed him to consider non-uniform stress distribution over the cross section of the bin. He derived his theory based on the assumption that all the inner rings slide downward relative to the outer ones. His solution shows the existence of two pressure regions, one in which the pressure increases with depth of grain and the other in which pressure is constant. The constant pressure value agrees with the limiting pressure predicted by Janssen's theory. His equations for filling (static) pressures are:

$$P_h = wKh(1 - u'K/D) \quad \text{for } h/D < 1/2u'K, \tag{4}$$

$$P_h = wD(4u')^{-1} \quad \text{for } h/D \geq 1/2u'K \tag{5}$$

Lvin's static lateral pressures are larger than Janssen's in the upper portions of the bin (Table I).

EMPTYING PRESSURES

General

It is generally accepted that wall pressures increase as deep bins are emptied. Building codes in some countries recognize this, and to account for it they usually suggest that pressures predicted by Janssen's theory be multiplied by a factor greater than one. In some codes the factor changes with bin depth. Although the factor is probably intended to allow for effects produced during emptying, it may also provide a factor of safety for other situations where uncertainty may occur. These could include vibration of a structure located near a railway track, swelling of grain during aeration or shrinkage of a bin during a sudden drop in ambient temperature. Most of the published studies are about pressure increases due to emptying through eccentric outlets, with some on thermal stresses. Studies related to other factors are scarce or non-existent.

Lvin's Theory

For pressures during emptying, Lvin's equation (Lvin 1970) reduces to:

$$P_h \text{ passive} = P_h \text{ active } (0.71 + 0.29\, m) \tag{6}$$

where m is the ratio of K passive to K active. He further states that this pressure increase during the transition from static to dynamic pressures occurs over a wall height of $H = 2D/2\acute{u}K(m+1)$ and that the pressure increase within this height is parabolic in shape.

Lvin's dynamic pressures are about two times the pressures predicted by Janssen's theory for $\phi = 27°$ and $\phi' = 17°$. Lvin's theory is not capable of predicting hopper wall pressures and the reduction in emptying pressures near the bottom of a bin as observed by many researchers including, Pieper (1969) and Deutsch and Schmidt (1969).

Jenike et al.

Jenike et al. (1973a, b) have given solutions for mass flow bins and funnel flow bins separately. The limiting conditions for mass flow can be determined from Fig. 1. Their solution is based on the principle of recoverable strain energy, but their equations to predict wall pressures are not easily understood. They have given their recommendations in graphical form for selected values of *h/D*. As read from their graphs, predicted pressures are 1.5 times the pressures predicted by Lvin's dynamic theory (Table I). In funnel flow bins, Jenike et al. (1973b) predict maximum pressures at the effective transition. This is the position at which transition from mass flow to funnel flow occurs, point *x* in Fig. 2(b).

THEORIES OF PRESSURES ON THE HOPPER WALLS

General

Very high pressures near the junction of a bin wall and hopper have been observed (Clague and Wright 1973). Many theories have been advanced to predict these hop-

per wall pressures. Theories of Walker (1966) and Walters (1973b) predict hopper wall pressures only for hoppers facilitating mass flow emptying of the bin (steep smooth hoppers). Clague and Wright (1973) have shown good agreement between their measured hopper wall pressures and pressures predicted by Walker's theory for mass flow hoppers. Theories of Everts et al. (1977) and Jenike et al. (1973a, b) predict hopper wall pressures for funnel flow and mass flow emptying of the bins. In funnel flow emptying (Fig. 2a), a central column of flow is formed above the discharge opening while most of the mass of grain remains almost static. The formed funnel progressively descends as the material is discharged at the bottom of the bin. During mass flow emptying (Fig. 2b), practically the entire mass of grain in the bin moves. Since grain bins usually have hoppers which produce funnel flow emptying, only theories dealing with hopper wall pressures in funnel flow bins are summarized below.

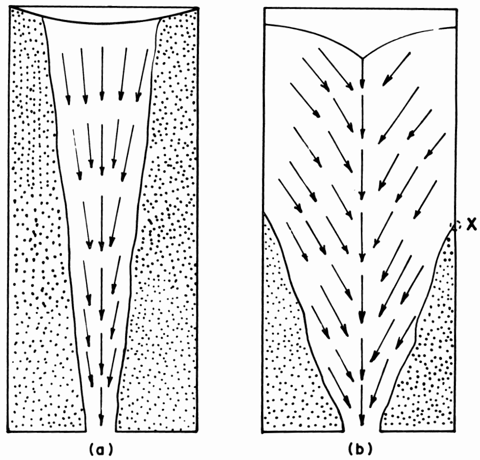

Figure 2. Emptying conditions in grain bins. (a) Funnel flow, (b) mass flow.

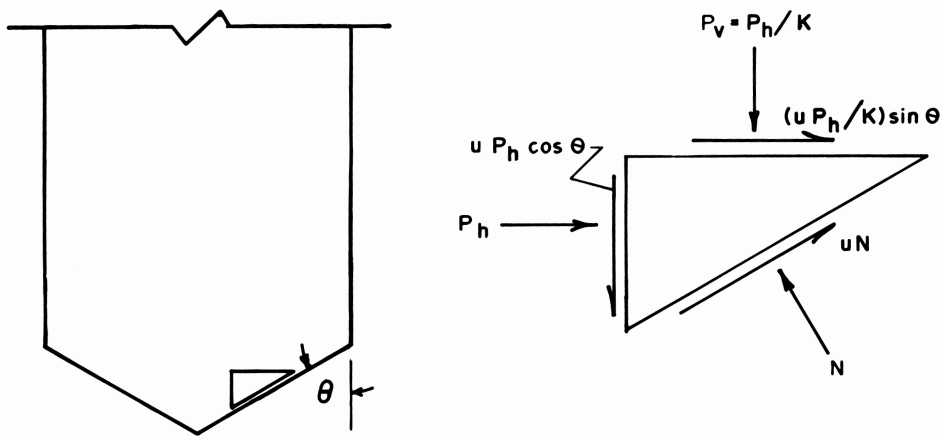

Figure 3. Location and equilibrium forces on a wedge of grain on hopper wall.

Funnel Flow Hoppers

By considering the dynamic equilibrium of a wedge of grain on a hopper wall (Fig. 3a, b), Jenike et al. (1973b) derived the following equations for normal hopper wall pressure at radius r from the central axis of the hopper:

$$N = P_h\left[\left(\frac{\sin^2\theta}{K} + \cos^2\theta\right) + \frac{2r}{D}u'(1 + 1/K \sin\theta \cos\theta)\right] \qquad (7)$$

The value of r varies from zero at the center to $D/2$ at the junction of the bin wall with the hopper. This gives a linear decrease in pressure from a maximum at the wall transition to zero at the apex.

To calculate hopper wall pressures at the junction, Everts et al. (1977) suggested the following modification to Eq. 7.

$$P_h\left[1.5\left(\left(\frac{\sin^2\theta}{1.5\,K} + \cos^2\theta + u'\left(1 + \frac{1}{1.5K}\sin\theta\cos\theta\right)\right)\right)\right] \qquad (8)$$

Moysey (1983) has shown results having good agreement with those calculated from Everts et al. (1977). For a typical hopper with 60-degree inclination to the vertical, and wheat stored in a plywood bin, the pressures near the junction of the bin wall to the hopper would be about three times the Janssen lateral pressure at that point.

Jenike et al. (1973b) also predict the pressure at the effective transition point in funnel flow bins (point x in Fig. 2b). This effective transition point could occur in a flat-bottomed bin or in one with a shallow hopper. The pressure at the effective transition point is given by:

$$P_t = \frac{K_t}{K}P_h \qquad (9)$$

where P_h is Janssen's lateral pressure at effective transition point, and

$$K_t = \frac{(24\tan\theta' + \pi/q)(1 - \sin\phi\tan\theta')}{16(\sin\phi + \tan\theta')} \qquad (10)$$

The plot for q (a constant) as a function of Θ and ϕ' for different values of ϕ are given in Jenike (1964). The values of Θ' as function of ϕ are given in Everts et al. (1977) and Troitsky (1980). For a particular value of ϕ, the values read for Θ' from the two sources are not the same. It seems that these plots are for two different values of ϕ', not mentioned on the graphs.

COMPARISON OF THEORETICAL TO EXPERIMENTAL RESULTS

Lateral pressures in bins during filling are shown in Figs. 4 and 5. Figure 4 is for a very deep bin, with a height to diameter ratio of 5, and compares two theoretical curves to Nielsen's (1982) measurements. The theoretical curves were calculated using $\phi = 30°$ and $\phi' = 25°$. Lvin's equation gives slightly larger values in the upper part of the bin, but below an h/D ratio of four the values are essentially identical. Nielsen's tests were done with barley in a silo 7 m in diameter by 46 m deep. The experimental curves clearly show that spouting the filling grain against one wall causes considerably greater pressure on that side of the bin than on the opposite side, particularly at the lower levels. Filling from a centric location reduced the pressure on side b but had little effect on the pressure on side a. Note that all measured values in the lower part of the bin are larger than the theoretical ones.

Figure 5 shows experimental data obtained by Moysey (1983) in a model bin where the maximum h/D ratio was 2.5, half the value of Fig. 4. These experiments were done with wheat in a plywood bin 0.7 m square by 2 m deep. Values of $\phi = 27°$ and $\phi' = 17°$, typical for wheat and plywood, were used to develop the theoretical curves. The bin was filled in a vertical stream from a funnel or in a sprinkling fashion from a conical spreader. The measured sprinkle-fill pressures are almost

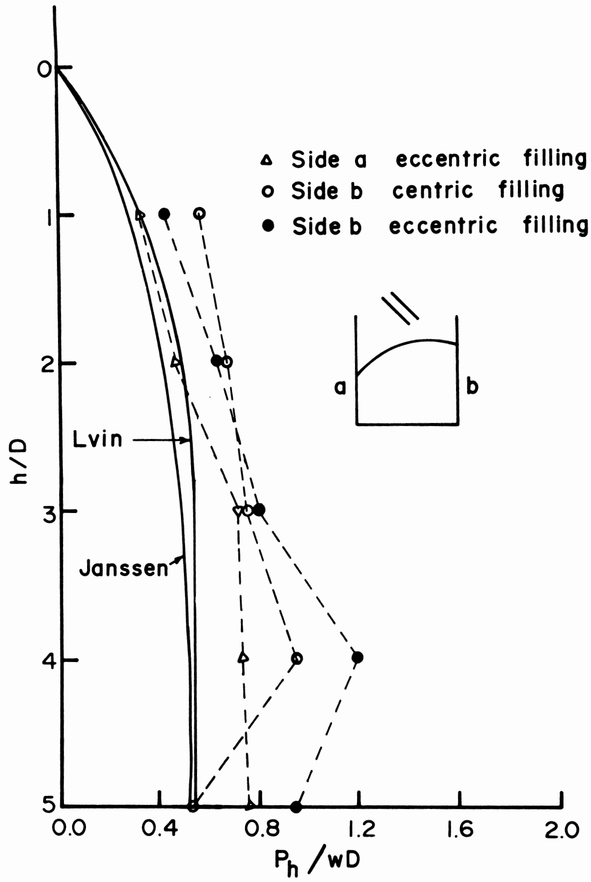

Figure 4. Theoretical static pressures for $\phi = 30°$ and $\phi' = 25°$ compared to Nielsen's results with eccentric filling.

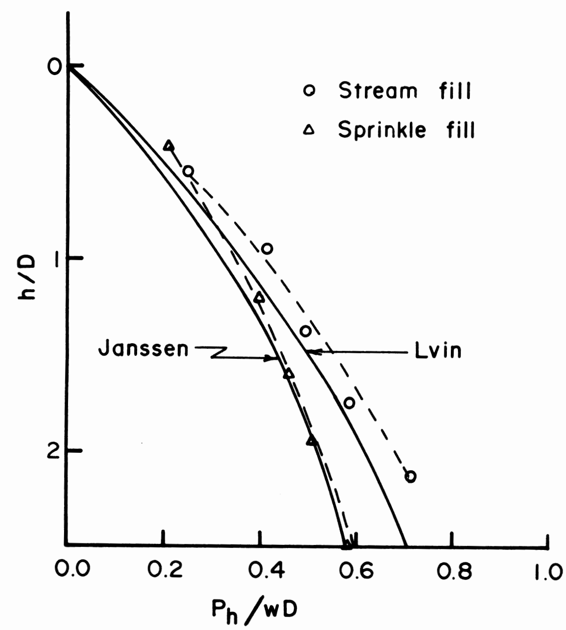

Figure 5. Theoretical static pressures for $\phi = 27°$ and $\phi' = 17°$ compared to Moysey's results.

identical to the calculated Janssen values. Static pressures resulting from a stream-fill process were appreciably larger than the pressures predicted by either Janssen's or Lvin's static theories. Previous work by a number of investigators has shown that the use of a grain spreader during filling increases the grain bulk density by 6 – 10%. One might expect this to produce larger pressures than a stream-fill, but a more dense fill means that particles are packed more closely, and possibly in some more orderly arrangement. Shear box tests (Moysey and Hiltz 1984) have shown that the apparent internal friction angle is larger for dense packing, which helps to explain the reduced wall pressures associated with it.

Figures 6(a) and (b) show the theoretical curves of Jenike et al. (1973b) and Lvin (1970) for pressures during emptying, along with the static pressure curves of Janssen for two sets of friction angles. As well, pressure measurements made by Kovtun and Platonov (see Turitzin (1963)) and by Pieper (1969) are given for comparison in Figs. 6(a) and (b), respectively. Kovtun and Platonov performed their tests with wheat in a full-scale concrete bin, whereas Pieper did his with malting barley in a model plywood bin 0.7 m square by 5 m deep. For both sets of friction angles, the Jenike curves give pressures that are much above measured values. Lvin's theory gives values that are slightly larger than the experimental results in both cases, but appear to give a much better approximation than Jenike's.

The Canadian Farm Building Code uses the Janssen equation to calculate wall pressures during filling and multiplying factors of 1.4 for centric emptying and 2.5 for eccentric emptying.

Figure 7 compares pressures perpendicular to hopper surfaces predicted by

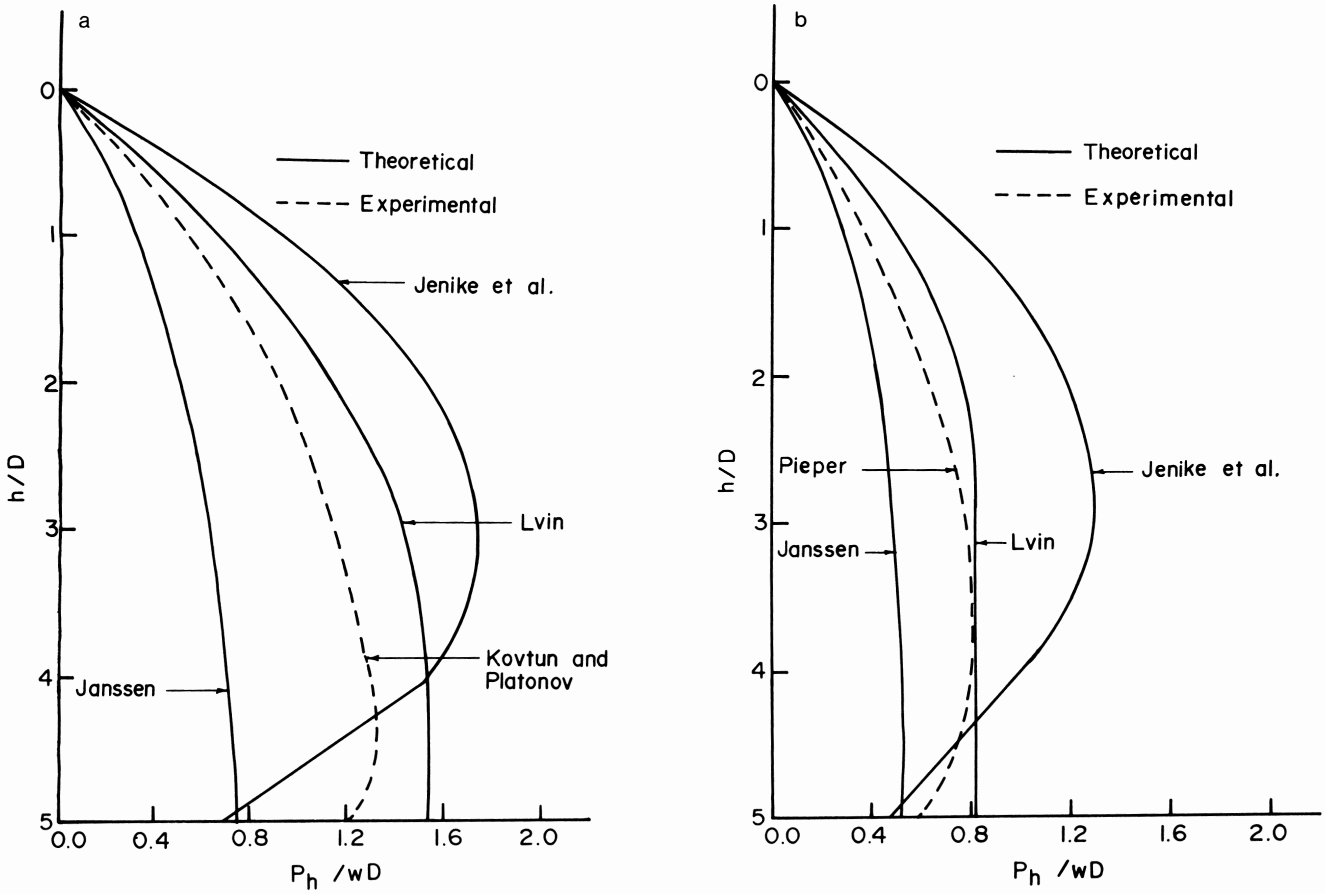

Figure 6. (a) Theoretical emptying pressures for ϕ = 27° and ϕ' = 17° compared with Janssen's pressure and experimental results of Kovtun and Platonov (see Turitzin 1963). (b) Theoretical emptying pressures for ϕ = 30° and ϕ' = 25° compared with Janssen's pressure and experimental results of Pieper (1969).

Jenike et al. (1973b) and by Everts et al. (1977) with pressures measured by Moysey (1983) using wheat in a model plywood bin. The curves and data are for a hopper sloped 60 degrees to the vertical. The experimental values for stream-fill are reasonably close to predicted values, but with sprinkle-fill they are substantially larger. Recalling that lateral pressures are low in the case of sprinkle-fill, it is not surprising that pressures on the bottom are large. Changing the hopper slope from 60 to 45 degrees reduces the experimental and calculated pressures slightly.

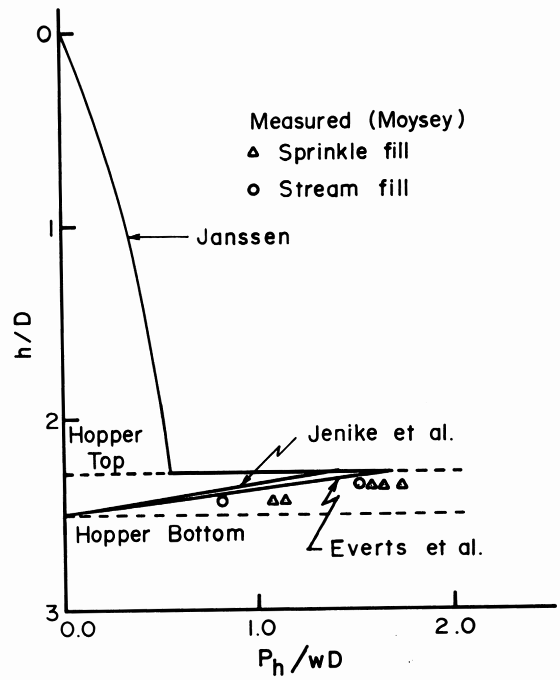

Figure 7. Theoretical and experimental pressures on hopper walls sloped 60 degrees to the vertical for ϕ = 27° and ϕ' = 17°.

SUMMARY

Pressures in large grain storages are affected by a number of factors, not all of which are considered in theoretical equations. Most models include wall friction angle, internal friction angle and bulk density, but make no provision for the effect of inlet spout location, the use of a grain spreader or the location of the discharge opening. The previous discussion has shown that:

(a) An inlet spout that directs grain against one wall of a bin can produce static pressures against that wall substantially greater than those predicted by Janssen's theory.

(b) Pressures during emptying can be considerably larger than under static conditions. Of the theories reviewed, Lvin's seems to fit the experimental data best.

(c) Using a grain spreader during filling reduces lateral wall pressures but increases

pressures on the hopper bottom.

(d) Measured pressures on hoppers were greater than those predicted by either of the theories reviewed.

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