Insulated sandwich wall construction is an excellent alternative to an insulated stud wall for livestock buildings and for insulated vegetable and fruit storage structures. Flexural stresses in sandwich walls are caused by lateral loads and by temperature differences of the wythes. The latter are the major concern of this paper. The results of a number of finite element stress analyses were used to determine thermally induced stresses for a variety of wall geometries and for different degrees of shear coupling between the concrete wythes. The stresses are presented in terms of the ratio of span length to overall wall thickness. The bending stresses due to temperature differences were found to be substantial under fairly normal Southern Ontario weather conditions. Tensile stresses of the order of 1 - 1.5 MPa are estimated to occur at the concrete-insulation interface.

### INTRODUCTION

Insulated concrete sandwich wall construction provides a good thermal barrier and in most cases has a reasonable heat storage capacity. Further advantages of this type of construction are its durability, and the fact that it does not support combustion. Furthermore, if well detailed, the construction is rodent-proof which is important for agricultural structures. A possible disadvantage of sandwich wall construction is that the determination of stresses due to flexure is difficult because of shear deformation in the insulation. As well, the type of connection between the two wythes of concrete affects the bending stresses considerably.

Tilt-up precast concrete sandwich panels have to withstand considerable bending stresses during erection when sections of wall that were constructed horizontally are lifted into a vertical position. Cast-in-place sandwich walls may be subjected to bending moments caused by lateral loads from backfill or from bulk materials stored inside the building. All sandwich walls will be subject to flexure caused by vastly differing temperatures of the two wythes.

Pfeifer and Hanson (1965) carried out a major experimental study at the Portland Cement Association laboratories. Some fifty 3 x 5-ft (0.9 x 1.5-m) sandwich panels with a wide variety of insulation and shear connector types were tested in bending using the largest dimension as the span length. They found that: (a) of the three metal shear connectors tested (truss type masonry reinforcing, expanding metal and welded wire fabric) only the two which had diagonal members significantly improved the structural behavior of the panels; welded wire fabric added very little to the bending rigidity; (b) comparison

![Graph](image)

Figure 1. Effective section modulus as a fraction of the fully composite section modulus vs. span length over panel thickness ratio for flexure due to a uniformly distributed loading.
<table>
<thead>
<tr>
<th>Series</th>
<th>Wythes Thickness</th>
<th>Core Thickness</th>
<th>Core Total</th>
<th>Span length</th>
<th>Span thickness ratio</th>
<th>Core thickness (%)</th>
<th>Core modulus (MPa)</th>
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\[ \text{DCA} = 0.182 + 0.0245 \frac{L}{t} \]  \hfill (1)

Figure 2. Qualitative bending stress diagrams.

Figure 1 provides a summary of their results.

Recent tests of sandwich panels carried out at the University of Waterloo confirm that truss-type shear ties add considerable shear strength to the core. The test panels were 184 mm thick with 50-mm-thick insulation. Spans ranged from 1.5 to 3.0 m. Although complete analysis of the results is not yet available, there is an indication that Eq. 1 provides a fair estimate for the calculation of maximum bending stresses.

Insulated sandwich walls are subjected to flexure by the differences in temperature of the two wythes as well. These thermally induced bending stresses are possibly more difficult to determine than those caused by external moments. A finite element study was employed to extend the earlier work into the area of thermal bending and the resulting stresses. The results of this study are the subject of this paper.

**ANALYSIS**

In an insulated concrete sandwich wall subjected to bending, plane sections do not remain plane. A qualitative picture of the bending behavior of a sandwich wall is shown in Fig. 2. Shear deformations take place in the core of the sandwich and the magnitude of these deformations depends on the ties provided between the two outer wythes. Solid concrete borders and intermediate concrete cross-connections provide an excellent shear connection between the wythes but at the cost of a considerable reduction in thermal resistance. Simple steel cross ties, on the other hand, do not cause much thermal "leakage" but also provide little in the way of shear resistance (Pfeifer and Hanson 1965). The use of metal ties with diagonal members such as is used for reinforcing masonry walls may well provide a good compromise. They are relatively cheap, readily available and they provide a reasonable amount of shear resistance.

Jofriet and Thompson (1984) found that the presence of metal truss-type ties can be modelled reasonably by increasing the stiffness of the core of the sandwich. They concluded that a modulus of elasticity of 20.7 MPa, three times that of the polystyrene insulation, provided maximum bending stresses that best approximated the experimental results. A similar approach will be followed herein and the results for different core stiffness will be examined.

Both the degree of composite action and the thickness of insulation core relative to the total wall thickness will have an effect on the bending stresses caused by differential temperatures. The modelling of the presence of metal truss-type ties by increasing the material stiffness of the core is approximate and based on tests of 127- and 152-mm-thick panels (see Fig. 1).
However, the effect of differing insulation (core) thicknesses will increase the shear deformations in the model and in the actual panel by about the same proportion.

The finite element method of analysis was used for the determination of stresses under thermal loading. The analyses were carried out on a predetermined temperature gradient from a steady-state heat transfer analysis using values of 2.2 and 0.028 W/(m.K) for the thermal conductivities of concrete and extruded polystyrene insulation, respectively. The presence of the shear ties was neglected in the heat transfer analysis.

All analyses were carried out for a 40°C difference in temperature between the outside faces of the wall. The heat transfer analysis showed that the difference in temperature between the outside face of the wall and the concrete insulation interface was only 0.33°C and 0.44°C for 76- and 51-mm insulation thicknesses, respectively. Thus, about 98% of the temperature difference is through the insulation and the concrete wythes have each almost constant temperature.

More accurate values for the thermal properties of sandwich panels may be obtained from a recent study by McCall (1985) who examined five different ways of coupling the concrete wythes and the effect this has on the overall thermal resistance of the panel.

The finite element analyses may be divided into five series. These are: I, 13 analyses of walls with a core stiffness of 20.7 MPa simulating truss-type shear ties between wythes; wall thickness and span length are the variables; II, four analyses of walls with the same core stiffness as in Series I but drastically different cross-sectional geometry; III, IV and V, three groups of 13 analyses equal to those of Series I but with varying core stiffnesses (6.9, 69 and 345 MPa).

The purpose of the analyses of Series I was to provide stress results for the type of construction favored in livestock housings (Jofriet and Singleton 1982, 1983). Truss-type shear ties are modelled here in a variety of cross-sectional geometries and differing span lengths. In Series II the objective was to check if the span length over thickness ratio is the governing parameter affecting the magnitude of the stresses.

In Series III, IV and V, the purpose of the analyses was to extend the work to other types of shear ties and to sandwich construction without shear ties. The latter would include the use of perpendicular through-ties which provide little shear resistance.

DISCUSSION OF RESULTS

The stress pattern caused by a typical temperature gradient in an insulated concrete sandwich wall is much like that induced by bending from transverse loads. The stress pattern found at the center of the length (or height), zero at the ends and vary as if the wall is subjected to a transverse uniformly distributed loading. In all analyses the walls were assumed to be free to rotate at the ends. This leads to bending stresses that are maximum at the center of the length (or height), zero at the ends and vary as if the wall is subjected to a transverse uniformly distributed loading.

Although there is a similarity between the temperature and transverse loading-induced stresses, there is the important parameter affecting the magnitude of the temperature difference between inside and outside faces; for that reason stress may be presented per °C. The results presented herein may be used for any temperature gradient between inside and outside by multiplying them by an appropriate temperature difference between inside and outside faces.

Details of the various finite element analyses are listed in Table I.

![Figure 3](image-url) Maximum bending stress per °C vs. span length over wall thickness ratio for all series I analyses/modulus of elasticity of core = 20.7 MPa.

![Figure 4](image-url) Maximum bending stress per °C vs. span length over wall thickness ratio for all series I and II analyses/modulus of elasticity of core = 20.7 MPa.
The results at the outside surfaces are indicated with open symbols; the inside surface stresses with solid ones. Stresses have been plotted versus the ratio of wall length to overall thickness.

It may be observed that for each of the three cross-sectional geometries analyzed the maximum stress increases nonlinearity with length. The rate of increase decreases as the length increases. It may also be observed that both the outside face and inside face maximum stresses are greatest for analyses (e) to (h) in which the core thickness is 27.6% of the total, and smallest for analyses (a) to (d) which has 41.3% core thickness. The stress results from analyses (i) to (m) with 32.5% core thickness lie in between.

Assuming that length to thickness ratio and core to total thickness ratio were the determining factors for the magnitude of stress, results from analyses with equal core to total thickness ratio were used to produce best fit relationships. These are shown in Fig. 4 — solid lines for the maximum stresses at the concrete insulation interface, chain-dotted lines for the maximum stresses at the outside faces. Furthermore, the results from the Series II analyses are indicated as well.

It may be seen from Fig. 4 that the results from analyses Id and Iib are identical although the geometries differ by a ratio of two. The same applies to analyses Ih and IId. But in both cases pairs of analyses have equal length to thickness and core to total thickness ratios. The results of analyses IIa and IIc have a length to thickness ratio that does not match any of the series I analyses; the results, however, match very closely the best fit curves from analyses I.

The results of analyses III, IV and V are shown in Figs. 5, 6 and 7. Again, results from analyses with equal core to total thickness ratios are connected with best-fit curves — solid for interior face stresses, chain-dotted for the outside face values. As with the results from analyses I, stresses increase with length to thickness ratio but less so as the core becomes stiffer. Also, the relationship between inside and outside face stresses is consistent.

A comparison of the maximum values of stress in Figs. 4, 5, 6 and 7 indicates that these maxima become greater as the core stiffness increases but not significantly so. At the largest length to thickness ratio (24.4) the maximum inside face stress increases from 0.043 MPa/°C at a core modulus of elasticity of 6.9 MPa to 0.051 MPa/°C at 340 MPa. The intermediate values are 0.050 MPa/°C at 21 MPa and 0.051 MPa/°C at 69 MPa.

For design it would be in order to accept the maximum value of 0.051 MPa/°C, regardless of the type of shear connection between the wythes. For strength design then the only factor remaining, besides temperature gradient, is the core thickness to total thickness ratio.

The variation of maximum stress with the core thickness to total thickness ratio is shown in Fig. 8. Both maximum stresses at the outside faces and at the concrete insulation interfaces are provided. The relationship between the maximum stress and core to total thickness ratio appears to be linear within the limits investigated in this work.

Using some sample dimensions a typical value for the maximum stress can be
calculated. A cast-in-place wall that is 225 mm thick with 75 mm insulation placed in the center would have a core to total thickness ratio of 33%. According to Fig. 8 the very maximum stress that could occur due to a temperature difference across the wall is 0.046 MPa/°C. In Southern Ontario a temperature difference between the faces of 30°C would not be unusual. This would result at most in a maximum bending stress of 30 × 0.046 = 1.4 MPa, a fairly high flexural stress.

A more detailed analysis requires more details. For instance, if the wall has truss-type shear reinforcement that can be modelled by a core with a modulus of elasticity of about 21 MPa, and if the wall is 3375 mm high supported top and bottom, Fig. 4 can be used. For a length to thickness ratio of 15 and core to total thickness ratio of about 33%, the maximum interior face stress can be estimated to be 0.039 MPa/°C. The maximum stress for a 30°C temperature difference therefore will be 1.2 MPa.

**SUMMARY AND CONCLUSIONS**

The bending stresses that may be expected to occur in insulated concrete sandwich wall construction because of the temperature differences between inside and outside wythes has been studied numerically. Fifty-six finite element analyses were carried out to investigate the effect of cross-sectional geometry, height to thickness ratio and of the degree of shear coupling between the two concrete wythes. It was found that: (a) the maximum bending stresses occur at the interface of concrete and insulation; (b) the bending stresses are highest in walls with the least thickness of insulation measured as a fraction of the total wall thickness; (c) the bending stresses increase with increasing height to thickness ratio if the modulus of elasticity of the core is small relative to that of the concrete; at high values of core stiffness the bending stresses are insensitive to changes in height to thickness ratio; (d) the maximum bending stresses decrease with increasing core to total thickness ratio; (e) bending stresses are somewhat higher in walls with more efficient shear coupling; (f) an upper-bound estimate to the temperature difference induced bending stresses can be made using the straight line relationships in Fig. 8 in which maximum bending stresses are plotted versus the ratio of core to total wall thickness.

**ACKNOWLEDGMENTS**

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**REFERENCES**


