Silage pressures at saturation in a tower silo

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Lau, A. and Jofriet, J.C. 1988. Silage pressures at saturation in a tower silo. Can. Agric. Eng. 30: 83-92. The structural safety of farm tower silos for whole-plant silage, and other cylindrical containers of wet materials that can become saturated in the lower part, necessitate a thorough knowledge of the liquid pressures that the silo wall has to withstand. The Canadian Farm Building Code has provisions for hydrostatic pressures in a farm tower silo, but the provisions are empirical and incomplete. This paper presents the results of a first attempt at using a finite element consolidation analysis of the fiber stresses and pore water pressures in the saturated zone of a body of silage. Each finite element analysis is preceded by a laminar analysis to determine the height of the saturated zone. The effects of the moisture content of the silage, its coefficient of permeability, the silo diameter and height and the bottom drain position were investigated in a parametric study. It was concluded that the excess hydrostatic pore water pressures were relatively small, and thus the normal hydrostatic pore water pressure is the most important component of liquid pressure at depth. An accurate determination of the height of the saturated zone, therefore, is essential. It could also be concluded that bottom drains should be placed near the wall. Finally, it was concluded that the finite element method can be a powerful research tool in solving this problem once the basic material properties are better known.

INTRODUCTION

The practice of storing whole plant material in tower silos as silage for on-farm livestock feed is now well established in Western Europe and North America. In favorable climates, such as the large corn growing areas of the midwestern United States, moisture contents of the silage are rarely greater than 65% on a wet basis. In northern parts of Europe and in Canada, however, moisture contents over 70% are more common. Although good quality silage can be produced over a wide range of moisture contents, those over 65% will cause saturation of the silage when the material surpasses a critical bulk density. This is very important for the determination of the structural loads on the silo wall because hydrostatic pressures are several times greater than those exerted by unsaturated silage.

Present tower silo structural design standards, codes and recommendations do not provide the flexibility to take into account properly the possible pore water pressure resulting from the saturation of wet silages. The Canadian Farm Building Code (National Research Council 1983) is one of the few standards that provides provisions for hydrostatic pressures in a farm silo in explicit form.

Lau (1983) studied the effects of migration of silage juice on the distribution and state of stress of the wet silage body. The overall objective of the research was to reduce the uncertainty associated with hydrostatic wall pressures in that zone of a tower silo where the silage is fully saturated. Detailed objectives toward achieving that overall goal were:

1. to determine the saturation level in the silage mass in a tower silo;
2. to quantify the instantaneous pore water pressure in the saturated zone and to simulate its time dependence;
3. to assess quantitatively the effect of fluid flow on the stress state of the silage material; and
4. to determine how various factors influence the stress distribution.

This paper presents some of the results of this research and their implications for the structural design of silos.

NOTATION

$C$, coefficient related to pore pressure increment (Nilsson)
$D$, silo diameter (m)
$E$, Young's modulus (kPa)
$E_s$, height of settled silage (m)
$H_s$, height of saturation zone (m)
$K$, lateral pressure ratio
$K_s$, components of permeability tensor (m$^4$/day·kN)
$K_v$, vertical pressure (kPa)
$L_x$, body force (m/s$^2$)
$e$, volumetric strain
$g$, acceleration of gravity (m/s$^2$)
$k$, coefficient of permeability (m/day)
$n$, outward normal
$M$, moisture content (wt %)
$P_{w}$, design lateral wall pressure at depth $z$ (kPa)
$P_{w}$, design lateral wall pressure at depth $E_s$ (kPa)
$q$, vertical pressure (kPa)
$q_v$, vertical pressure at saturation (kPa)
$P_{sat}$, saturation density (kg/m$^3$)
$p$, bulk density (kg/m$^3$)
$p_{sat}$, saturated density (kg/m$^3$)
$p_{juice}$, density of juice (kg/m$^3$)
$P_{juice}$, density of juice (kg/m$^3$)
$P_{wall}$, density of wall (kg/m$^3$)
$v$, Poisson's ratio
$z$, depth of silage below settled surface (m)
$z$, depth of silage above settled surface (m)
$\mu$, coefficient of friction between silage and silo wall
$\tau$, excess pore water pressure (kPa)
$\sigma$, coefficient of permeability (m/day)
$\phi$, coefficient of friction between silage and silo wall
$\phi$, coefficient of friction between silage and silo wall
$\iota$, differentiation with respect to time

BACKGROUND RESEARCH

A comprehensive study of lateral pressures exerted on silo walls by whole-plant silage material was carried out by Wood (1970) who obtained an empirical pressure-density-time relationship. He suggested that effective saturation of the silage occurs when about 10% of the total silage volume remains as gas (air) and that the saturation density, $p_{sat}$, is dependent on the moisture content.
content, \( M \):

\[
\rho_{\text{sat}} = 1440 - 5.4 M
\]  

(1)

in which \( \rho_{\text{sat}} \) is in kPa and \( M \) is in percent.

In the saturation zone, he assumed that the fiber contact stress (the effective stress transmitted through the silage medium from particle to particle) remains constant with depth. This means that the increase in pore pressure for each layer is equal to the increase in total vertical stress. Wood assumed it to vary linearly with depth below the saturation level. The total lateral pressure is then equal to the sum of the pore water pressure and the fiber stress at that depth.

't Hart et al. (1979) followed up on Wood's work and further investigated the physical properties of silage up to a moisture content of 74%. He also adopted the 10% gas volume assumption for effective saturation. However, he derived the saturation density from fundamental physical relationships as follows:

\[
\rho_{\text{sat}} = \frac{1440}{(1 + 0.006 M)}
\]  

(2)

which yielded results similar to those obtained by Wood in the region of 60–80% moisture content. Their results differ by 3.5–5.5% from those of Wood (1970).

't Hart et al. (1979) instrumented a 6.19-m-diameter steel silo with pressure-measuring panels, and recorded pore water pressure as well as the total lateral wall pressure. Their experimental results, using a 60% moisture content grass silage, indicated that a maximum pore water pressure of 23 kPa was developed at 2 m above the silo base 7 days after filling. The measured pore water pressures were found to make up 50–80% of the total lateral pressure, thus demonstrating the significance of pore water pressure on the estimation of the lateral wall pressure.

Another research project related to the present study was carried out by Nilsson (1982) in Sweden. To cope with Scandinavian conditions, he worked with grass silage with moisture contents up to 85%. Nilsson introduced two concepts in connection with the criterion of saturation. He defined an "apparent saturation" and an "effective saturation" stage. Nilsson hypothesized that drainage would take place only after the effective saturation stage had been attained.

For a watertight silo, Nilsson specified for the increment in pore water pressure

\[
\Delta p = C \Delta q \quad 0 < C < 1.0
\]  

(3)

at the apparent saturation stage, and

\[
\Delta p = \rho_g g z
\]  

(4)

after effective saturation occurs.

Another new concept was Nilsson's use of "moisture density", the difference between wet and dry bulk densities, as a means of determining saturation. Juice was suggested to be released first at a constant value of this moisture density, rather than at a fixed gas content.

A number of design codes and standards deal with farm tower silos. Some of the more relevant ones are the Canadian Farm Building Code (National Research Council 1983), the British Standards Institution (1974) and the standards published by the International Silo Association (1982a,b).

The Canadian Farm Building Code (National Research Council 1983) provides an empirical formula for lateral pressure for concrete silos and for whole-plant silages not exceeding 65% moisture content (wet basis).

\[
\rho_{\text{uns}} = 4.8 + 0.58 HD^{0.55}
\]  

(5)

For wet silage, it further specifies the total lateral pressure of silage plus its juice at any depth below the saturation level as:

\[
\rho_{\text{uns}} = 4.8 + 0.58 H D^{0.55} + (9.81 - p_d/D)(z - H_d)
\]  

(6)

Equation 6 consists of three terms. The first two represent the fiber stress between the saturation level and the bottom of the silo, and the third the hydrostatic pore water pressure. The Canadian Farm Building Code does not provide precise guidelines for determining \( H_d \). Rough estimates are given as 30 m for 65%, 16 m for 70% and 11 m for 75% moisture contents.

The British Standards Institution BS 5061 (1974) gives a lateral pressure expression for two cases:

\[
\rho_{\text{uns}} = 9.8 + (9.8 - 29.5/D)(H - 3) \quad H > 3 \text{ m}
\]  

(7)

for silos in which there is no intentional provision for the release of excessive fluid pressure and:

\[
\rho_{\text{uns}} = 9.8 + 0.75 (9.8 - 29.5/D)(H - 3) \quad H > 3 \text{ m}
\]  

(8)

for silos in which drainage for excessive fluid is provided.

Both equations are for a depth greater than 3 m below the silage surface. A uniform lateral pressure of 9.8 kPa is recommended above that level.

It is apparent from these equations that emphasis has been placed on the occurrence of hydrostatic pore water pressure, even in well-drained silos where a factor of 0.75 is used to reduce the term containing hydrostatic pore water pressure. The term 29.5/D appears to implicitly account for material properties such as friction and moisture content.

In the United States, the most recent set of standards is published by the International Silo Association (1982a,b). However, they are intended only for storing-silages with a maximum moisture content of 65%. These new standards adopt Wood’s laminar approach for analysis, and give recommended pressures based on Janssen’s equation (1895).

\[
\rho_{\text{uns}} = (pD/\mu)(1 - \exp((-4 Kz/D)))
\]  

(9)

**METHOD OF ANALYSIS**

The study comprised two parts: (1) simulation of the sequential filling of silage; and (2) its consolidation behavior upon saturation. A laminar approach was used to simulate the sequential filling to determine the saturation level and the surcharge load at that level. Both are inputs to the second part in which a series of finite element analysis was carried out to conduct a preliminary investigation into the consolidation behavior.

The following assumptions were made:

1. Effective saturation is achieved when 10% of the silage medium remains as air (Wood 1970; 't Hart 1979).
2. In the saturation zone, hydrostatic pore water pressure and pore water pressure in excess of the hydrostatic may be developed.
3. Unsaturated silage above the saturation zone is treated as a surcharge imposing a uniformly distributed load on the material below applied instantly at time of saturation.
4. Juice flow is associated with the saturated zone only, while the contribution of flow from the unsaturated zone is neglected.
5. After primary consolidation, as determined in the simple laminar approach, further consolidation of the saturated zone does not change the density and elastic properties appreciably.
6. The moisture content of the saturated silage remains constant during consolidation.
7. Wall friction below the saturation level is negligible.
Figure 1. Typical saturated zone with boundary conditions.

The laminar analysis was used to determine the depth of saturation, \( H_s \). The saturation criteria developed by 't Hart (Eq. 3) was used in this analysis. The laminar analysis is simply an iterative solution for the equilibrium of vertical forces in thin lamina of silage, assuming that a silo is filled by adding thin layers on top of those already present. The analysis incorporates a vertical pressure-density model (Jofriet et al. 1982) to simulate primary consolidation before saturation. After saturation, the silage is assumed to become incompressible in this first stage of analysis. The parameters considered in the laminar approach were moisture content, lateral pressure ratio and wall friction coefficient, as well as silo diameter.

After the height of the saturated zone was determined in the laminar analysis, the saturated part of the silage body was analyzed further using the finite element method.

The finite element formulation developed by Sandhu (1968) was based on Biot’s (1941) theory for the consolidation of a saturated porous medium. In brief, the coupling equation governing continuum displacement and fluid flow is in the form of Darcy’s law for irrotational flow:

\[
-e_a = K_a (\sigma_{ij} + p_w \delta_{ij}), \quad i
\]

The discretisation procedure outlined by Sandhu for a two-dimensional plane problem was modified by Lau (1983) to suit the axisymmetric nature of the silage problem in a cylindrical tower silo.

The primary output from a numerical analysis of this kind are nodal displacements and excess hydrostatic pore water pressure. The secondary outputs are the derivatives in the form of effective stresses and velocities. The velocities, of course, can be used to determine quantities of flow.

In a typical cylindrical silo problem, two sets of boundary conditions are allocated to each node in the grid, one related to the displacements and the other related to flow, as shown in Fig. 1.

Assuming a rigid wall, all boundary nodes except those along the top are constrained at right angles to the boundary. Similarly, the velocity normal to boundaries BC, AE and CE is zero except at the drain where the hydrostatic pore water pressure is zero. This condition is also assumed to exist along top boundary AB above which lies the unsaturated zone.

The analysis can deal with anisotropic mechanical and flow properties of the stored material. Each element can be assigned different properties from those of adjacent elements. Consequently, it provides much flexibility in dealing with difficult materials such as silage which are likely to be anisotropic.

Depending on when steady-state with respect to consolidation was reached, the total time span of an analysis ranged from 5 to 15 days. Lateral wall pressures, which are of immediate concern to wall design, were computed by summing their three components: excess hydrostatic pore water pressure set up by the surcharge load, hydrostatic pore water pressure due to saturation and fiber contact stress associated with the wall nodes.

First, one typical case of haylage stored in a “watertight” tower silo (analysis PS 001) was studied in depth. The finite element layout is shown in Fig. 2. Local mesh refinement was made where the steepest pressure gradients caused by excess hydrostatic pore water pressures were anticipated. The following are the parameters:

(i) material: 100% alfalfa haylage harvested at the early stage

(ii) properties

- moisture content, \( M = 70\% \)
- density, \( \rho \_{\text{sat}} = 1014 \text{ kg/m}^3 \)
- Young’s modulus, \( E = 300 \text{ kPa} \)
- coefficient of permeability, \( k = 0.4 \text{ m/day} \)
- Poisson’s ratio, \( \nu = 0.23 \).

(iii) silo geometry

- silo diameter, \( D = 6.1 \text{ m} \); silo height, \( H = 24.4 \text{ m} \)
- drain position: 100-mm-diameter perimeter drain, located 0.5 m from the wall on the floor
(iv) wall friction coefficient $\times$ lateral pressure ratio, $\mu K = 0.2$

Moisture content ($M = 70\%$) marks the approximate threshold value for distinction between "dry" and "wet" silages. Young's modulus ($E = 300$ kPa) was obtained from the expression (LeLievre and Jofriet, 1982)

$$E = 3.52 q$$  \hspace{1cm} (11)

The coefficient of permeability $k$ was found to vary inversely with moisture content and lateral pressure ratio. Poisson's ratio ($\nu = 0.23$) corresponds to that of an isotropic elastic material having a lateral pressure rate of 0.3 (LeLievre and Jofriet 1982). Finally, $\mu K = 0.2$ is a representative value for a steel silo (Jofriet et al. 1981).

In addition, a parametric study was performed to study the effect of a number of parameters. Table I provides a list of the analyses carried out and demonstrates the extent of the parametric study.

**RESULTS AND DISCUSSION OF LAMINAR ANALYSES**

Table II lists the depth of saturation, height of saturated continuum, and saturation density, along with magnitude of the vertical pressure at the depth of saturation for various values of silo diameter, moisture content, and product of wall friction coefficient and lateral pressure ratio, $\mu K$.

For the analysis PS 001, the depth of saturation, $H_d$, was calculated as 13.5 m below the surface of the settled silage, when the silage body had undergone a settlement of 9.5 m during the filling process. This value is in line with other researchers' works. Nilsson (1982) estimated the depth of saturation to be 16 m in his full-scale experimental work with 62% moisture content grass silage in a 6-m-diameter steel silo. Wood (1970) found that silage with a higher moisture content would reach saturation at a shallower depth.

$H_d$ was found to vary inversely with moisture content and silo diameter, as expected. Considering a $\mu K$ of 0.2, $H_d$ decreases from 11.4 m for $M = 70\%$ to 4.3 m for 75% in a 7.3-m-diameter silo. For silo diameters of 5.5 m and 9.1 m, $H_d$ is 15.6 m and 10.2 m, respectively, for $M = 70\%$.

Wall friction and the lateral pressure ratio have a significant influence on $H_d$. This is especially so with smaller diameters (below 7.3 m) and lower moisture contents (below 75%). Again using $M = 70\%$, $H_d$ increases from 11.4 m for a $\mu K$ of 0.2 to 21.4 m for a $\mu K$ of 0.3 when $D$ is 7.3 m. However, it only varies slightly from 10.2 to 13.5 m when $D$ becomes 9.1 m.

The results of the analyses indicate that saturation is not likely in 24.4-m-high silos if $\mu K$ is assumed to be 0.3, if the diameter is less than 7 m and the moisture content is 70% or less.

Vertical pressures at the saturation level were found to be practically the same for all silo diameters considered, independent of $\mu K$. Yet, it varied considerably with moisture content, being about 100 kPa for $M = 67\%$, but only 50 kPa for $M = 75\%$. This allows for a simple empirical expression for estimating saturation depth from vertical pressure in the silage. The approximate relationship is:

$$q_v = 600 - 7.4 M$$  \hspace{1cm} (13)

The laminar analysis described herein provides conservative saturation depths, because the time factor is not taken into account in the application of the load and in the consolidation process. In a real situation, filling of a silo (applying the load) often takes a week, and full consolidation takes a month or longer. Thus, drainage can take place before the load has reached its maximum level and before the material is fully consolidated.

**RESULTS AND DISCUSSION OF ANALYSIS PS 001**

The profiles of excess hydrostatic pore water pressure along the wall and on the central axis at $t = 0$ (transient response) and at $t = 7$ days (steady-state) are plotted in Fig. 3. Along the wall, the maximum excess hydrostatic pore water pressure is initially found about 7.5 m above the base. The top boundary separating the unsaturated and saturated zones was treated as perfectly permeable and the excess pore pressure therefore is essentially zero. It is noted that the maximum pressure of 23 kPa is only about 27% of the magnitude of the surcharge load of 85 kPa. This indicates that for non-zero lateral strain, the total applied traction is partly resisted or carried by the solids and partly absorbed by the pore water (Craig 1978). Excess

<table>
<thead>
<tr>
<th>Analysis no.</th>
<th>Silo size $D \times H$ (m x m)</th>
<th>M (%)</th>
<th>Drain location†</th>
<th>$k_v$ (m/day)</th>
<th>$k_t$ (m/day)</th>
<th>$E$ (kPa)</th>
<th>$\nu$</th>
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<tr>
<td>PS 001</td>
<td>6.1 x 24.4</td>
<td>70</td>
<td>1</td>
<td>0.4</td>
<td>0.4</td>
<td>300</td>
<td>0.23</td>
</tr>
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<td>PS 002</td>
<td>6.1 x 24.4</td>
<td>67</td>
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<td>0.4</td>
<td>0.4</td>
<td>300</td>
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<tr>
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<td>75</td>
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<td>0.23</td>
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<tr>
<td>PS 001</td>
<td>6.1 x 24.4</td>
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<td>0.4</td>
<td>0.4</td>
<td>300</td>
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<tr>
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<td>0.4</td>
<td>0.4</td>
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</tr>
<tr>
<td>PS 006</td>
<td>6.1 x 24.4</td>
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<td>1</td>
<td>0.004</td>
<td>0.004</td>
<td>300</td>
<td>0.23</td>
</tr>
<tr>
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<td>6.1 x 24.4</td>
<td>70</td>
<td>1</td>
<td>0.4</td>
<td>0.4</td>
<td>300</td>
<td>0.23</td>
</tr>
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<td>6.1 x 24.4</td>
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<td>1</td>
<td>4.0</td>
<td>4.0</td>
<td>300</td>
<td>0.23</td>
</tr>
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<td>6.1 x 24.4</td>
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</tr>
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<td>0.23</td>
</tr>
<tr>
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<td>300</td>
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<td>300</td>
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†Drain location: 1, perimeter drain 2, center drain.
Table II. Results of lamina analysis

<table>
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<tr>
<th>$M$</th>
<th>$\rho_s$</th>
<th>$H_u$</th>
<th>$H_i$</th>
<th>$H_a$</th>
<th>$\mu K$</th>
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<tr>
<td></td>
<td></td>
<td>5.5m</td>
<td>6.1m</td>
<td>7.3m</td>
<td>9.1m</td>
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<tr>
<td>57%</td>
<td>1027 kg/m$^3$</td>
<td>15.6 m</td>
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<td>11.4 m</td>
<td>10.2 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.9 m</td>
<td>9.6 m</td>
<td>11.7 m</td>
<td>12.7 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.5 m</td>
<td>6.1 m</td>
<td>7.3 m</td>
<td>9.1 m</td>
</tr>
<tr>
<td>70%</td>
<td>1014 kg/m$^3$</td>
<td>11.4 m</td>
<td>11.4 m</td>
<td>11.4 m</td>
<td>11.4 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.5 kPa</td>
<td>99 kPa</td>
<td>102 kPa</td>
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<tr>
<td></td>
<td></td>
<td>18.6 m</td>
<td>13.0 m</td>
<td>10.1 m</td>
<td>8.4 m</td>
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<tr>
<td>72%</td>
<td>1006 kg/m$^3$</td>
<td>10.2 m</td>
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<tr>
<td></td>
<td></td>
<td>64 kPa</td>
<td>67 kPa</td>
<td>68 kPa</td>
<td>68 kPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.8 m</td>
<td>11.4 m</td>
<td>14.3 m</td>
<td>16.0 m</td>
</tr>
<tr>
<td>75%</td>
<td>993 kg/m$^3$</td>
<td>8.8 m</td>
<td>9.6 m</td>
<td>11.7 m</td>
<td>14.2 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.1 m</td>
<td>7.1 m</td>
<td>7.1 m</td>
<td>7.1 m</td>
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</table>

$\mu K = 0.2$ and $0.3$

$D$ is the diameter of the silo.

Figure 3. Excess hydrostatic pore water pressures along the axis are generally higher than those along the wall because this region is comparatively farther away from the drain. A maximum value of 32 kPa occurs at the bottom, yet, it still represents only 38% of the surcharge pressure.

The magnitude of radial displacements is in the order of $10^{-3}$ to $10^{-5}$ m and is therefore negligible. However, vertical displacements are much larger. Total settlement of the saturated zone, obtained by the summation of nodal vertical displacements was 0.7 m at 7 days. This represents 6.5% of the height of the saturated zone and shows that a saturated silage mass is not very compressible, even with a gradual release of silage juice.

The total lateral pressure in the saturated zone initially and after 7 days is illustrated in Fig. 4. A comparison was made with the design formula (Eq. 6) in the Canadian Farm Building Code. The British Standards Institution (1974) prediction (Eq. 7) is also shown in Fig. 4. The maximum total pressure at the silo bottom obtained numerically is about 65% greater than the Canadian Farm Building Code value, and nearly identical to that suggested in BS 5061. It should be emphasized again that the total pressure depends strongly on the conservatively estimated saturation level. Also, assumptions 2, 6 and 7 tend to provide conservative numerical results at this time.
RESULTS AND DISCUSSION OF THE PARAMETRIC STUDY

Results of the parametric study will be discussed relative to the effects of the parameters moisture content, drain position, silo diameter, coefficient of permeability and silo height.

Effect of moisture content (Analyses PS 001, PS 002, PS 003)

Moisture contents of 67, 70 and 75% were considered in a 6.1 x 24.4-m silo filled with silage having an isotropic coefficient of permeability of 0.4 m/day. Figure 5 illustrates the effect of moisture content on excess hydrostatic pore water pressure. The general shapes of the three hydrostatic pore water pressure curves are similar, but the maximum values are somewhat different, ranging between 20 and 25 kPa.

For 75% moisture content, the total lateral pressure increased to a maximum after 2 days due to the increase in excess pore pressure. The total wall pressure profile is maximum at 0 for moisture contents of 67% and 70%. Of course, a higher moisture content leads to larger total lateral wall pressures. The maximum total lateral wall pressure was 77 kPa for 67% silage, whereas the 75% moisture content material produced a value of 193 kPa.

Effect of drain position (Analyses PS 001, PS 010)

In Fig. 6, initial excess pore pressures at the wall are plotted for a perimeter (wall) drain and a center-drain position. Excess pore pressures are seen to increase with depth for the latter situation. This shape is expected because of the impervious nature of the boundary near the wall. Comparing the results in Fig. 6 with those in Fig. 3 shows that excess pore pressures at the wall nodes for a center-drain are greater than those at the centerline for a perimeter drain. It is evident that a center-drain is less effective for reducing excess pore pressures in the model, since the area of the free boundary at that drain is much reduced.

Figure 7 shows the effect of drain position on the total wall pressure. Naturally, the total wall pressure is higher for a center-drain. At the base, it reaches a maximum of about 190 kPa in 5 days' time, compared with a maximum of 115 kPa attained at $t=0$ for the case of a perimeter drain. This 65% increase is a very significant finding, reinforcing the recommendation of a perimeter drain as one simple way to avoid excessive build-up of hydrostatic pore water pressures in a watertight silo.

Effect of silo diameter (Analyses PS 001, PS 004, PS 005)

Figure 8 shows the excess hydrostatic pore water pressure profiles for silo diameters of 6.1, 7.3 and 9.1-m. At $t=0$, the upper half of the curves indicate large excess hydrostatic pore water pressure gradients of up to 19 kPa/m for the 7.3- and 9.1-m-diameter silos. These gradients either reinforce or counteract the gravitational influence, depending on their signs. The laminar analyses indicated that the surcharge load is insensitive to changes of silo diameter. Apart from giving rise to a slightly deeper saturated zone, a larger diameter also induces a greater portion of the surcharge load to be spread out laterally, thus weakening its influence on the bottom region. In general, greater total lateral wall pressures result from a larger diameter.
Effect of permeability (Analyses PS 001, PS 006, PS 007)
The coefficient of permeability (isotropic) was varied from 0.004 m/day to 4.0 m/day. Numerical analysis results indicate that as the coefficient gets smaller, the maximum momentary excess hydrostatic pore water pressure may reach 66 kPa at 0.004 m/day compared to 23 kPa at 0.4 m/day, as illustrated in Fig. 9. At the lowest value, the excess hydrostatic pore water pressure is approximately 80% of the surcharge load, and thereby reduces the effective stress. This considerable increase in pore water pressure naturally leads to higher lateral wall pressures in the upper part of the saturated region, while near the drain the pressure is much less affected.

Effect of silo height (Analyses PS 001, PS 008, PS 009)
The excess hydrostatic pore water pressure profiles for the selected silo heights of 18.4, 21.4 and 24.4 m are very similar. Therefore, Fig. 3 is representative of the excess hydrostatic pore water pressures that prevail. Since the surcharge loads on the continua are equal in magnitude for all cases, the consistency in these profiles and the mode of subsequent pressure dissipation is expected.

Combined pressure profiles
The complete lateral wall pressure profiles (unsaturated and saturated zones) are shown in Figs. 10, 11 and 12 demonstrating the effects of moisture content, silo diameter and silo height. It is noted that these are maximum pressures attained at different times, as indicated on each curve. For comparison, the design formula recommended by the Canadian Farm Building Code (Eq. 6) has been included. An examination of Figs. 10,
silo size = 6.1 x 24.4 m
M = 70%

Figure 9. Excess hydrostatic pore water pressures for three values of isotropic permeability at $t=0$.

SUMMARY AND CONCLUSIONS
A first attempt at a numerical study of the migration of silage juice in the lower part of a tower silo has been presented. The study employed the finite element method which provided effective stresses due to the silage and hydrostatic pressure from the silage juice separately.

The finite element method can provide a valuable tool for the analysis of the consolidation behavior of saturated silage. Generally, the adaptation of the axisymmetric finite element analysis in conjunction with the laminar approach for finding the extent of the saturated zone was reasonably successful in achieving the stated objectives.

Findings from the study are summarized as follows:

1. The depth of saturation varied nonlinearly with moisture content, silo diameter, wall friction coefficient and lateral pressure ratio. It varied approximately linearly with vertical stress (Eq. 12). Accordingly, the saturation level varied from 20 to 80% of the silo height. Such a large variation will lead to vastly different hydrostatic pressures in a fully saturated mass.

2. The excess hydrostatic pore water pressures were relatively small, and thus the lateral wall pressure was largely dependent on the hydrostatic component, which in turn was very sensitive to the saturation level. Hence, there is some urgency to reduce the disparities among criteria now available.

3. The magnitude of the hydrostatic part of the lateral wall pressure is most sensitive to moisture content. Silo diameter...
and drain position are also important influences, while silo height had the least effect.

(4) The permeability has a marked short-term effect on the excess hydrostatic pore water pressure, especially in the upper part of the saturated zone. The coefficient of permeability of 0.4 m/day used for most analyses was selected on the basis of a limited number of consolidation tests of alfalfa haylage. There is evidence that in large silos permeabilities much lower than 0.4 m/day exist near the bottom of the silo.

(5) A steady state with respect to consolidation was reached in 2-11 days, depending on various parameters. No general relationship was established.

The laminar approach itself is valuable in that it is simple and capable of providing reasonable results with respect to saturation. However, the uncertainty regarding the criteria for the saturation density prohibits direct use of these results for making recommendations on tower silo design loads. 't Hart's saturation criterion seems to be the best available for structural design, since excess hydrostatic pore water pressure can only develop when saturation is reached. As was pointed out, the results presented herein are likely to be conservative because of neglecting certain practical time factors in finding the saturation depth.

However, one recommendation can be made with certainty. Tower silo wall pressures can be reduced substantially if the hydrostatic pore water pressure can be relieved quickly by effective drainage. This is accomplished in a concrete stave silo through the many joints in the wall. In a cast-in-place concrete silo, effective drainage is more difficult to achieve. Bottom drains provide fast relief near the drain, but a reduction further away takes time. In the field, many such structures have vertical cracks because greater-than-expected lateral pressure causes the tensile strength of the concrete to be exceeded. After cracking, the silo has more effective drainage, but then there is a risk of steel corrosion.

It is premature to use the present results for structural design purposes. A further investigation will have to take into account that loading is not instantaneous, and that drainage will likely start before full loading is reached. Also, wall friction in the saturation zone needs to be considered. Both factors will tend to reduce the maximum lateral pressures that will occur in a real silo. However, this first attempt has helped to identify the important factors and the need for increased knowledge of some basic properties of silages.

To allow further work in this area, basic material properties such as a permeability-density model and better elastic or pseudo-elastic properties are needed. Other silages such as corn silage need to be investigated as well.

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