Trailing vortex instability and its implications for aerial spraying

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INTRODUCTION

Aerial spraying differs from ground based pesticide application in two ways. First, the height of the spray boom above the crop is higher in aerial spraying and second, the air motion induced by the passage of the aircraft affects the spray dispersal. It is generally recognized that these factors are responsible for the greater potential for spray drift in aerial spraying. Consequently, a considerable amount of work has been expended in modelling and understanding aircraft wakes in an attempt to develop techniques that will minimize off-target drift of pesticides.

Starting with Reed (1954), modelling of the effect of the aircraft wake on spray dispersal has been achieved at various levels of complexity. A good review of this modelling effort can be found in Green (1987). In the more elaborate models, the aircraft wake is allowed to develop in a turbulent boundary layer flow and the only mechanism for decay is by eddy-viscosity effect and turbulent diffusion. More abrupt forms of decay resulting from instabilities are ignored. However, the motion of small spray droplets, which are potentially entrained in the vortex flow (Drummond 1987), depends largely on the fate of the vortices. Therefore, it is essential that vortex decay should be properly accounted for when assessing drift potential in aerial spraying.

The scope of the work presented here encompasses an evaluation of the importance of vortex stability as a mechanism of vortex decay for a small, clean aircraft in steady flight near the ground. A series of full scale experiments will be described. In these experiments, the cores of the tip vortices were marked with smoke and their motions followed in space allowing unstable vortices as well as the type of instability to be clearly identified. A detailed account of the experiments and further results on vortex motion are given in Drummond et al. (1987, 1992).

REVIEW OF LITERATURE

Aircraft trailing vortices can become unstable by two mechanisms: sinusuous instability and core bursting. The sinusuous instability was first described analytically by Crow (1970) and is often referred to as "Crow instability". The analysis by Crow applies out of ground effect in a neutrally stable atmosphere. In this type of instability, the vortices from each wing tip undergo symmetric sinusuous motion leading to linking of the two vortices followed by the formation of ring vortices which subsequently decay. The most unstable mode leading to linking is symmetric with a wavelength equal to 8.6 times the wing span.

Core bursting, on the other hand, is poorly understood. Review articles by Hall (1985) and Leibovich (1983, 1984) conclude that there is not yet a satisfactory explanation of the mechanism. Tombach et al. (1977) described core bursting in trailing vortices where apparently there is a sudden conversion of rotational energy into axial flow resulting in a swift and localized destruction of the organized vortical flow. This description of core bursting is, however, a somewhat oversimplification of the situation as we note that Faler and Leibovich (1977) classified six different types of vortex core
bursting. Further, Tombach et al. (1977) stated that not even the definition of core bursting is agreed upon. Escudier (1986) discussed experimental and theoretical efforts on vortex breakdown and concluded that there was no general agreement on the essential facets of the flow such as stagnation, wave phenomena and instability. Also, Escudier stated that none of the theories attempts to describe quantitatively all details of breakdown. Most experimental work relies on flow visualization because it has been found that the presence of instruments or sensors in the vortical flow introduces distortions leading to erroneous conclusions. Lissaman et al. (1973) discussed the stability of aircraft trailing vortices and noted that the core bursting phenomenon was not related to interaction between the vortices trailing from each wing tip but rather to the development of the core itself. They defined core bursting as the onset of axial flow in the vortex core which is the first instant where there is a noticeable gap in the smoke used to mark the core. This definition will be the one used in this report and it is illustrated in Fig. 1. Tombach et al. (1977) summarized most of the available data on vortex bursting and concluded there was a general paucity of full scale experimental data for small aircraft in steady flight near the ground. They also developed a model for vortex breakdown, correlating the time-to-burst of a vortex with atmospheric turbulence. That result is used in this report.

EXPERIMENTAL PROCEDURES

The test site was located in open country near Navan, ON, where the terrain was flat for 5 km in all direction. In adjacent areas, the ground was covered with grass, typically 0.6-1.0 m tall, except over the experimental site where the grass was mowed regularly. Of the four possible flight paths over the test site, the one selected for an experimental run had to be within 30 degrees to the perpendicular to the wind vector. With the aircraft flying over the outer boundary of the test site and the wind blowing towards the center of the site, the vortices were thus always moving towards the center away from the boundary under the action of the wind. The atmospheric boundary layer flow upwind of the flight paths was unobstructed and the surface roughness was virtually the same for winds from any experimentally useful direction.

A Harvard Mark IV aircraft (Fig. 2) was used in the tests. The Harvard has a length of 8.8 m, a span of 12.8 m, a wing area of 24.0 m² and a takeoff mass of 2450 kg. The nominal flight speed was 57 m/s for all tests and nominal altitudes were 3.0, 7.6, and 15.2 m simulating both agricultural and forestry spraying. The altitude of the aircraft was taken to be the vertical distance between the ground and the propeller spinner. The aircraft was equipped with a full-span, faired spray boom located just behind and below the trailing edge of the wing (see Fig. 2). At takeoff mass and a speed of 57 m/s, the aircraft generated tip vortices with a circulation strength of 34 m/s. The values of both the wing span and the vortex strength compare well with those of a Cessna Agtruck, a typical aerial spraying aircraft, for which the span is 12.7 m and the vortex strength is 31 m/s at normal flight speed and mass.

Smoke cans were installed underneath the wing tips of the aircraft at 30% chord from the leading edge to mark the vortex cores. The cans were approximately 250 mm in length with a diameter of approximately 100 mm with a closest approach to the skin of about 30 mm. The long dimension was aligned with the wing chord. Electrical ignition caused a dye to burn for about 20 seconds leaving 1.2 km long smoke trails behind the aircraft. The smoke left stains over the central half chord of the wing tip and was thus clearly injected directly into the vortex core and not just wrapped around it. Tombach (1973) showed pictures of almost identical smoke residues deposited on the wing tip from smoke grenades installed inside the wing tip.

A helicopter-based camera and another one on the ground photographed the passage of the aircraft and its trailing vortex system. The helicopter, a Bell 47, hovered at 250 m above the flight path. In the first part of the experimental program, the pilot was aided in his positioning by an array of colored patio stones laid into the ground. Win this system, station keeping proved to be difficult and a down-pointing video camera connected to a display in the cockpit was installed to make the task simpler for the latter portion of the program. This was a valuable addition since video pictures could be recorded as a backup.
Fig. 2. The Harvard aircraft with the spray boom attached.

to the airborne camera. This camera used 70 mm film and operated at 8 frames per second with a 150 by 150 m field of view on the ground. The ground-based 16 mm camera, with a 40 m field of view, was positioned about 160 m downwind from the flight path and used to measure aircraft speed and altitude. A frame rate of 100 frames per second was required to record unblurred images of the aircraft. Both Dee and Nicholas (1968) and Tombach et al. (1977) used similar camera arrays.

The time-to-burst is defined as the time of the start of axial flow observed from the 70 mm film. This film was chosen in preference to the 16 mm film because of its wider field of view and better photographic contrast. The film was advanced until the first break in the smoke trail marking the core was observed. Then, the film was reversed until the aircraft was at the same place as this first break, the number of frames moved being recorded. Dividing the number of frames by the frame rate yielded the time-to-burst.

Instrumentation on the ground recorded the meteorological data that were used in conjunction with the camera-recorded vortex data. In the first part of the experimental program, two sonic anemometers were mounted on a tower at 5 and 10 m above ground near the center of the test site and for the last part of the program a third sonic anemometer was installed at 20 m above ground. The anemometers are described in Drummond and Barszczewski (1985). Meteorological data were collected for 30 minutes centered around the time of the aircraft passage over the test site at a rate of 20 samples/s. These records were used to compute the mean wind speed and direction as well as power spectral densities. From these spectral densities, the turbulent dissipation rates were calculated using atmospheric surface layer theory in the inertial subrange (Panofsky and Dutton 1984).

RESULTS

Table I summarizes the experimental conditions. For all experiments, the aircraft speed was nearly the same with a mean value of 55.4 m/s. The height of the aircraft above the ground varied between 2.8 and 19.2 m. In Table I, both wind-speed and turbulent dissipation rate \( \varepsilon \) are given at the height of the aircraft. The exponent of a power law profile was calculated from the measured windspeed and then applied to obtain the windspeed at aircraft altitude. The turbulent dissipation rate \( \varepsilon \) was obtained at the aircraft height \( z \) by extrapolation from each measurement point using:

\[
\varepsilon = \frac{\varepsilon_1}{z}
\]

where \( \varepsilon_1 \) is the measured turbulent dissipation rate at an anemometer height \( z_1 \) (Lissaman et al. 1973). Using Eq. 1, estimates of the dissipation rate at the aircraft height from each anemometer were obtained. As these generally agreed within 10% for any given test, they were averaged to yield a single value. The range of windspeed obtained in the field test covers the range of interest in aerial spraying with a minimum of 0.36 m/s and a maximum of 4.48 m/s. Turbulent dissipation rates varied between 0.34 and 482.2 cm/s². In the last column of Table I, instances where vortex stability appeared to be influenced by external devices are identified. One of these devices was a small triangular antenna near the port wing tip (Fig. 3) whose presence hereafter will be referred to as the antenna case. The other device was a pair of ground markers for use as a reference for the pilot and the ground-based camera which will be referred to as the marker case. The two markers were tarpaulins about 1 m high and 2 m wide stretched between 50 by 50 mm wooden stakes and oriented perpendicular to the flight path about 7.6 m on either side of the aircraft track.

In the present series of tests, Crow instability was never observed. Whenever sinuous motion appeared, it was generally asymmetric and no tendency for the two vortices to link occurred. The vortex system always became unstable by core bursting which usually followed some asymmetric sinuous motion. Figure 1 is a typical view from the helicopter-based camera and it clearly shows this type of motion. The variation in height of the vortices along their axes can be inferred by close examination of the shadows of the smoke trails on the ground.

The time-to-burst was measured for all the tests. Figure 4 compares time-to-burst for the upwind vortex, \( T_{bu} \), to time-to-burst for the downwind vortex, \( T_{bd} \), for the no-antenna and no-marker cases. The regression line was computed on the assumption that the standard deviation of the error increases linearly with \( T_{bd} \) (Snedecor and Cochran 1978). Evidence for this assumption came from a regression analysis of the absolute residual error as a function of \( T_{bd} \) as recommended in
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*Underlined = antenna location
Brown (1980). The regression coefficient was found to be significantly different from 1 at the 95% confidence level \( t=2.7, 18 \text{ d.f.} \). This implies that on average the downwind vortex bursts first. The explanation for this trend can probably be traced to the interaction of the wind shear with the vortical flow. Time-to-burst varied between 2.5 and 42.0 s for the upwind vortex and between 3.4 and 37.5 s for the downwind vortex.

Time-to-burst data for the marker case are presented in Fig. 5. Again, the regression coefficient is significantly different from 1 at the 95% confidence level \( t=5.8, 6 \text{ d.f.} \). Chow's test (Brown 1980) was performed to compare the regression coefficient of the marker case to the one of the no-marker, no-antenna case. At the 95% confidence level, the test revealed that the coefficients are not significantly different \( \text{F}=2.7; 1.24 \text{ d.f.} \). This result was not expected since a close examination of the helicopter-based film showed that in all cases, the downwind vortex burst at the marker even if it had burst before. Moreover, no obvious effect of the marker can be identified for the upwind vortex. In the marker case, the time-to-burst for the downwind vortex varied between 2.75 and 6.0 s and for the upwind vortex between 5.63 and 8.5 s.

A regression analysis was also applied to the data for the antenna case. In contrast to the marker case, the time-to-burst for the starboard vortex is plotted against the time-to-burst for the port vortex since the antenna was under the port wing which was not always downwind (Fig. 6). The regression coefficient is significantly different from 1 at the 95% confidence level \( t=5.92, 14 \text{ d.f.} \) and is also significantly different from the coefficient for Fig. 4 at the 95% confidence level \( \text{F}=10.8; 1.32 \text{ d.f.} \) where there is no antenna. This leads to the conclusion that the presence of the antenna induced early bursting and this occurred even though the port wing was upwind in 5 tests out of 15. It is conjectured that the triangular antenna acted as a small delta wing and shed a locally strong vortex that wrapped itself around the core of the trailing vortex and triggered the core bursting by some unknown mechanism. The evidence is circumstantial at best but this explanation is consistent with the statement of Donaldson and Bilanin (1975) that "wake development is extremely sensitive to configuration details of the lift, drag and thrust producing mechanisms of the aircraft."

As time progresses, a vortex may suffer from multiple bursting and/or decay under the action of turbulence and interaction with the ground plane. As a result, at some time following the passage of the aircraft, there is no longer any discernable organized flow on the aerial photographs and this time defined the extinction of the vortex. A measurement of the time-to-extinction was made for each vortex, but it is not precise and required some judgment because sometimes faint streaks of smoke remained visible as vestiges of a vortex core but no solid segments were apparent. Time-to-extinction data are plotted on Fig. 7 as a function of time-to-burst, although in four tests, extinction could not be read from the film. The data show that the maximum time interval between extinction and the first onset of axial flow was 18 s but the interval was less than 10 s for fully 3/4 of the cases. On average, the time delay between bursting and extinction was the same for the downwind and upwind vortices and it was not affected by the presence of the antenna or the marker. The regression line in Fig. 7 is a reasonable fit to the data with \( R^2=0.81 \).
Tombach et al. (1977) showed one case of experimental data where the burst vortex passed through an anemometer which measured a velocity history consistent with vortical flow with a marked reduction in maximum velocity. This indicated that there was still some rotational flow in the vortex even after core bursting. It was impossible to investigate this conclusion during the tests reported here because the anemometer tower was erected too far from the line of flight. However, in some cases, a thin filament of smoke remained after a burst, which indicates that the core was not always totally destroyed. This partly supports the theoretical conclusions of Khorrami (1991) that unstable disturbances are traveling outside the core and that most of the energy in the unstable modes is distributed outside the core, leaving the core intact. Also, in at least one instance in the current set of experiments, there was considerable downwind transport (125 m) of a "slice" of smoke and/or spray after core bursting had occurred and the slice was observed to retain significant rotation.

**COMPARISON WITH A MODEL**

Tombach et al. (1977) developed a model for predicting time-to-burst and their result is repeated here:

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In the atmosphere, the constant $\beta$ can be neglected because the turbulent dissipation rate is typically at least an order of magnitude larger. Substituting the value of $b$ in centimeters and neglecting $\beta$ leads to:

$$\frac{19.6}{\varepsilon^{1/3}} < T_b < \frac{78.6}{\varepsilon^{1/3}}$$

when the factor of 2 accuracy is taken into account. Equation 3 includes the effect of the aircraft height as can be deduced by combining Eqs. 1 and 3.

Figure 8 presents time-to-burst versus dissipation rate data for the no-marker, no-antenna cases. Both the downwind and upwind vortices share a common lower bound. The upper bound for the downwind vortex is slightly lower than the one for the upwind vortex, but the difference is small compared to the accuracy of Eq. 3 and a single upper bound was retained. The upper and lower bounds of the data compare quite well with Eq. 3. Further, Tombach (1973) presented data having the same upper and lower bounds as those in Fig. 8 from a series of experiments using a Cessna 170 aircraft out of ground effect. It should be noted that, the maximum vortex strength for the Cessna 170 was $33 \text{ m}^2/\text{s}$ compared to a value of $34 \text{ m}^2/\text{s}$ for the Harvard used here. Thus, even if Eq. 3 were shown to apply both in and out of ground effect, questions of scale on the time-to-burst results remain unresolved.

The agreement between the present data on time-to-burst (Fig. 8) and that of Tombach (1973) supports the contention that the smoke cans did not affect the vortex stability. In the data from Tombach, the smoke-generating grenades were mounted inside the wing tips while for the Harvard, the smoke cans were external. The direction of the air motion around the wing tip changes from almost completely span-wise under the wing to almost completely vertical at the edge downwind vortices. Close to the ground in the atmosphere, the constant $\beta$ can be neglected because the turbulent dissipation rate is typically at least an order of magnitude larger. Substituting the value of $b$ in centimeters and neglecting $\beta$ leads to:

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$$\frac{19.6}{\varepsilon^{1/3}} < T_b < \frac{78.6}{\varepsilon^{1/3}}$$

when the factor of 2 accuracy is taken into account. Equation 3 includes the effect of the aircraft height as can be deduced by combining Eqs. 1 and 3.
of the wing tip allowing smooth flow to occur around the smoke can. Chordwise components of the flow cause a small wake behind the can but there is no vorticity in such a flow.

Figures 9 and 10 present data for the marker and antenna cases, respectively. As expected, the presence of the antenna or the marker promoted early bursting of the affected vortex. In the marker case, both vortices were affected.

**IMPLICATIONS OF RESULTS FOR AERIAL SPRAYING**

First, the time scale of the vortex core bursting phenomenon should be compared to the time scale of the spray deposition process. Calculations by Drummond (1992) using vortex trajectory data collected during the early portion of the experiments (Numbers 1 to 26, Table 1) showed that more than 90% of the emitted spray was still airborne at the end of the vortex lifetime. It should be noted that the aircraft height was relatively large (6 to 19 m above ground) for this portion of the experiments compared to the height for the latter portion (nominally 3 m). In those early experiments simulating forestry spraying, the volume median diameter of the droplet distribution varied between 110 and 120 μm and the measured volume on the ground varied from 20% to 60% of that sprayed by the aircraft. In the calculations, the volume emitted from the aircraft was chosen to be equal to the ground deposit in each case even though the ground deposit accountability was often low. The calculations showed that only the large drops (greater than about 220 μm) deposited on the ground while the vortices remained intact. It thus appears that the time scale of bursting is significantly shorter than that for spray deposition. This conclusion should be checked for agricultural simulations because a larger drop spectrum is usually emitted closer to the ground but vortex core bursting also occurs earlier as can be inferred from Eqs. 1 and 2.

In both areas of aerial spraying, the abrupt transition from vortex-dominated to atmospheric-dominated droplet transport after vortex extinction must be carefully considered. It is important to know when and why to switch processes while modelling spray transport. Otherwise, large differences can be found between predicted and measured ground deposition.

The effect of vortex bursting on ground deposition is not known. However, since the vortex affects the initial spray behavior, it is expected that significant variations in ground deposit along the aircraft path can be induced by core bursting. The strong axial flow observed to take place in either direction away from the point of core bursting could serve to collect the spray between points of instability. The analysis of Khorrami (1991) tends to support this because he calculated a surprisingly large axial flow outside the core for an unstable mode, the region where a large portion of the spray is located.

An additional perplexing issue is that after the burst, the state of the vortex is not even qualitatively understood. Persistence of a swirling flow has been reported here for one case and Tombach (1973) reported a similar event.

Present models of aerial spray dispersion do not include vortex bursting. However, the data presented above clearly show that core bursting is the mechanism of vortex decay. This conclusion is unlikely to be specific to any particular aircraft because vortex bursting has been observed for aircraft of such diverse size as a Cessna 170 and a Boeing 747.

Modellers should at least include an approximate treatment of this phenomenon. As a first step, one could compare the evolution of the spray cloud when the vortex system is said to disappear after time-to-burst has elapsed, to the evolution of the same cloud when bursting is ignored. Unfortunately, this appears to be the limit for modelling until more theoretical and experimental work is performed.

**CONCLUSION**

The vortex system of the Harvard aircraft in ground effect always decayed by core bursting and the vortex downwind of the flight path generally burst first. The time-to-burst measurements were in good agreement with the correlations of Tombach (1973) except in two cases. The first exception occurred when a ground marker was observed to trigger an early vortex burst. The second involved a small triangular antenna mounted under the port wing tip which induced early bursting of that vortex. After bursting, vortex annihilation occurred within 18 s. The main conclusion is that vortex core bursting is the mechanism of vortex decay for a small clean aircraft cruising in ground effect and that this phenomenon should influence spray dispersal. In the present state of knowledge, this influence cannot be quantified.

**ACKNOWLEDGEMENT**

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