Structural design of liquid manure tanks

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Jofriet, J., Zhang, Y., Johnson, J.W. and Bird, N. 1995. Structural design of liquid manure tanks. Can. Agric. Eng. 38:045-052. Liquid manure is commonly stored in reinforced concrete cylindrical tanks when land application is not possible. The structural design of liquid manure tanks requires a consideration of all loads and of appropriate support conditions. The most important loads for which a cylindrical liquid manure tank must be designed are reviewed. The selection of an appropriate support condition at the bottom of the tank wall is discussed. The assumption of a radial spring is recommended for tanks in which floor and wall are not monolithic. The determination of the internal hoop forces in the wall of a cylindrical liquid manure tank is difficult if the boundary support at the bottom of the wall is other than sliding or hinged. Design curves are provided to aid designers to determine the hoop tension in the wall and in the footing. As well, the radial displacements can be determined using the same design curves. Friction and passive earth pressure can be considered to provide the horizontal support at the wall footing. The frictional resistance is easy to calculate; the passive earth pressure can only be estimated. An example analysis of the hoop tension in a liquid manure tank is provided in the Appendix.

INTRODUCTION

Many farm operators use a manure storage facility to be able to apply manure to the land mainly during the vegetative season. Many of these storage facilities are built of reinforced concrete and are large enough to store the manure produced over six to ten months. Steel tanks, although common in Europe, have not been used extensively in North America. The determination of tank size is described in detail in the Canadian Farm Buildings Handbook (Anonymous 1988).

Liquid manure tanks must be designed to adequately withstand all loads. As well, a serviceable tank should be watertight to prevent the pollution of groundwater and the corrosion of the reinforcing steel. In Canada, the National Building Code (NBC) (CCBFC 1995b) and the Canadian Farm Building Code (CFBC) (CCBFC 1995a) have provisions that are relevant to the design and construction of farm manure storage structures. Some provinces, such as Ontario, have their own building code in place of the NBC (e.g. OBC 1993) but all provincial building codes are virtually identical to the NBC where structural design is concerned (i.e. Part 4).

Most free-standing liquid manure tanks are cylindrical in shape because evenly distributed loads on a cylindrical wall create almost no bending moments as compared to rectangular walls. A straight wall subjected to the same load is more expensive to build and may require buttresses at regular intervals for stability. A rectangular tank can serve as the foundation for a building with slatted floors. In this circumstance, concrete cross-beams or a floor will serve as support for the wall and replace the buttresses.

In most circular liquid manure tanks, the major structural component is the tank wall which, with its footing, is often constructed separately from the tank floor. Thus, the bottom of the wall can move out radially under the effect of the internal pressure and create a mostly undetermined support condition at the bottom of the wall.

The objective of this paper is to review the various loads that a cylindrical tank wall must be able to resist and to address the question of support condition that might be appropriate at the bottom of the wall. A design aid for determining hoop tensions is presented and an example is included to illustrate its use.

TANK WALL LOADS

The following are the loads, other than dead load, that must be considered in the design of a liquid manure tank wall. They are specified in the 1995 CFBC (CCFBC 1995a) and/or Part 4 of the 1995 NBC (CCFBC 1995b). It has been assumed that the tank wall is vertical:

1. Outward horizontal liquid pressure from the manure at an equivalent fluid density of 10 kN/m$^3$. The tank should be assumed filled to the top of the wall unless there is a positive overflow device preventing this from happening.

2. Inward horizontal soil pressures based on the equivalent fluid densities listed in Table I. Below the assumed watertable, the equivalent fluid density for the first...
three materials in Table I may be taken as 50% of those shown. Of course, the water pressure (see point 3) below the watertable must be added to the soil pressure. The equivalent fluid density of poorly drained soft silts and clays includes the groundwater pressure.

3. Inward hydraulic pressure from groundwater increasing with depth below the highest known watertable at the rate of 9.8 kN/m$^2$ per metre depth. If there is uncertainty about the highest watertable, it is prudent to assume it to be at the level of the local undisturbed ground level (exclusive of any berming). This could be reduced if granular backfill and a perimeter drainage system is used.

4. Uplift on all parts of a tank below the assumed design watertable. If a manure tank can be emptied during periods of high watertable, it is necessary to provide some means of relieving the water pressure. This can best be accomplished by providing a compacted granular fill base for the floor and a sub-floor drainage system tied to a perimeter drain. As an added measure of safety, a pressure relief plug can be built in the tank floor. If this plug does pop up, care should be taken to ensure that the plug has been properly reseated or replaced before the tank is put back into service.

5. The NBC and the CFBC indicate that where the backfill may be subject to vehicular loads within 1.5 m of a manure tank wall, such as manure tankers or trucks, the walls must be designed for a horizontal surcharge load of 5 kPa applied uniformly below ground level.

6. Weight of the manure on the footing and/or floor (density 10 kN/m$^3$).

7. Uniform snow and rain loads if the liquid manure tank is covered. Snow and rain loads will depend on location (CCBFC 1995b). If a storage tank cover is not exposed to vehicular traffic it must be designed for the dead plus snow loads, or dead load plus 2.0 kPa, whichever is greater (CCBFC 1995b). If the tank cover may receive vehicular traffic, the minimum uniformly distributed live load for farm machinery traffic is 7.0 kPa (CCBFC 1995a). Where it is anticipated that the cover will be occupied by either loaded trailers, trucks, or farm tractors having a mass in excess of 6000 kg (including the mass of mounted equipment), the live load must not be less than 10 kPa (CCBFC 1995a). Alternatively, a concentrated wheel load of 23 kN per wheel, applied over an area of 750 mm by 750 mm located so as to cause maximum effects must be considered. Where a tank cover serves as a place for processing or for loading or unloading of vehicles, live loads must be increased by 50% to allow for impact or vibration of the machinery or equipment (CCBFC 1995a).

8. Forces caused by expansion of ice that may form on the surface of the manure. Although the 1995 CFBC specifies that ice loading must be designed for in areas of the country where ice forms in liquid manure tanks, there is little guidance as to the magnitude of loading except for climates similar to that of the Quebec City area (CCBFC 1995a; Jofriet et al. 1996).

9. Temperature gradients in the tank wall and creep and shrinkage of the concrete.

**LOAD COMBINATIONS**

When designing a tank wall, the following major live load combinations must be considered:

1. Outward pressure of the manure on the wall plus ice pressure. The inward soil loading on the wall may not be used to offset the manure pressure unless thoroughly compacted granular backfill is used (CCBFC 1995a). Cohesive soils when used for backfill tend to shrink away from the wall making their support of the manure pressure for the life of the structure uncertain. If compacted granular backfill is used, the maximum resistance that should be subtracted from the manure pressure is a low estimate of the active pressure. Inward groundwater pressure should never be used to offset manure pressure.

2. Inward pressure on an empty tank resulting from a combination of groundwater pressure, the soil backfill, and the surcharge due to vehicles travelling within 1.5 m of the wall.

In both load combinations 1. and 2., other loads must be considered such as dead loads, live load reactions on the wall from a possible tank cover, temperature variations in the structure and structural components, creep, and shrinkage of the concrete. In load combination 2., uneven levels of backfill must be taken into account. This is very difficult to analyze and the authors recommend that backfill levels be kept the same all around a cylindrical tank both during construction and afterwards.

**INTERNAL FORCE ANALYSIS**

The inward soil pressure from the backfill and the groundwater pressure will cause a circumferential compression in a cylindrical tank wall. Providing these loads are uniform around the tank perimeter, the hoop (circumferential) compression will be uniform. For concrete tanks these hoop compression forces and the resulting hoop compressive stresses are small compared to the compressive strength of concrete. For instance, for a 50 m diameter, 2.5 m deep tank backfilled with a poorly drained soft silt or clay, the maximum compressive stress will be about 2 MPa. For a wall made of 30 MPa concrete this is not excessive providing the inward pressure is approximately uniform around the circum-
ference of the tank. If not, the wall may buckle much like a column or similar compression member.

In the case where a tank wall is supported at the top by a cover as well as at the bottom by the floor, the wall will function mainly as a vertical beam spanning between roof and footing. The hoop tension will be negligible. The determination of the bending moments from the outward manure pressure and the inward soil/groundwater pressure is simple and does not deserve further discussion.

The majority of tanks do not have a cover and these need to be considered in more detail. The outward manure pressure causes circumferential tension in the cylindrical wall which creates stress in reinforcing steel bars and to a certain extent in the concrete (Jofriet et al. 1987). An accurate estimate of the hoop tension in the wall and its distribution over the height is essential. Once the hoop tension is determined, the hoop reinforcing steel and the wall thickness can be determined (Jofriet et al. 1992).

The hoop tension is a function of the outward pressure on the wall and the support at the bottom. If the floor of the tank and the wall footing form a homogeneous unit as is common practice in Québec (Fig. 1), and if the floor is suitably reinforced to withstand the radial reaction from the wall, a hinged support can be assumed. The hoop tension resulting from the manure pressure can then be determined with the aid of the appropriate table in PCA (1993).

![Fig. 1. Cross section of liquid manure tank wall integral with floor slab.](image1)

A common construction method in Ontario uses a joint between the wall and its footing and the tank floor allowing these to separate (Fig. 2). This type of construction allows the construction of the floor after the wall and unforeseen uplift pressures underneath the tank only affect the floor rather than the entire tank.

Some (e.g. Mullen 1981) have assumed that the wall can slide freely at the bottom so that the hoop tension at each level in the wall equals the pressure at that level multiplied by the tank radius and the radial reaction at the bottom of the wall is zero. This assumption is not accurate because:

1. There is always some horizontal reaction at the bottom of the footing from friction.

2. The footing at the bottom of the wall is larger and hence stiffer than the wall providing additional resistance to free radial displacement.

An assumption that the joint is hinged with zero radial displacement, on the other hand, is not realistic either if the construction procedure is as indicated in Fig. 2 or if the floor in Fig. 1 is not reinforced and likely to crack. The "real" boundary condition therefore lies somewhere between a hinged joint and a perfectly sliding support.

The horizontal reaction from friction is a function of the vertical force in the wall from the dead load and the downward friction from the backfill. The radial reaction may be as high as 50 to 75% of the vertical force.

The effect of the footing stiffness exceeding that of the wall is equivalent to a radial spring support at the bottom of the wall (see Fig. 2). Assuming that there is a radial reaction force, $F_r$ (N/m), at the wall-footing interface (see Fig. 3), the hoop tension in the footing, $T_f$, is:

$$T_f = \frac{F_r D}{2} \tag{1}$$

in which $D$ is the tank diameter. The hoop tensile stress in the footing can be obtained by dividing the hoop tensile force by the footing area, $A_f$, and the hoop strain in the footing, $\varepsilon_h$, can be calculated, assuming linear elastic behaviour, as:

$$\varepsilon_h = \frac{F_r D}{2 A_f E_c} \tag{2}$$

where $E_c$ is Young's modulus of concrete. The radial displacement, $\Delta R$, of the footing, and of the bottom of the wall, due to $F_r$ alone is the hoop strain multiplied by the tank radius or:

$$\Delta R = \frac{F_r D^2}{4 A_f E_c} \tag{3}$$

If the footing is represented by a linear spring with stiffness $k'$ (N/m per m circumference), then a radial reaction $F_r$ would result in a radial displacement:
The hoop tension in a wall with height $H$ supported at the bottom by a radial spring was determined by means of a number of linear elastic axisymmetric finite element analyses. The wall was subjected to a triangular hydraulic load (liquid weight $w$ kN/m$^3$) increasing from zero at the top of the wall to $wH$ kN/m$^2$ at the bottom. For each tank geometry, four spring stiffnesses were considered.

In all tanks, except those with a sliding base, the magnitude and distribution of the hoop tension is based on tank geometry. The tank geometry can be expressed by the non-dimensional geometry ratio $H^2/(D\ell)$, where $\ell$ is the uniform thickness of the wall. Large diameter shallow tanks have a small $H^2/(D\ell)$ ratio. The smallest value considered was 0.4 (e.g. a 50 m diameter by 2.45 m deep tank with a 300 mm thick wall). Deep tanks with a relatively small diameter have a large value of $H^2/(D\ell)$; in this study 8.0 was the largest value used (e.g. a 10 m diameter tank, 4.5 m deep with a 250 mm thick wall). Intermediate values were 0.8, 1.2, 1.6, 2.0, 4.0, and 6.0.

The standard tank designs provided by the Ontario Ministry of Agriculture, Food and Rural Affairs (Jofriet 1992) have 1.5$t$ mm thick footings that project $t$ mm beyond the faces of the tank wall. This provides a footing with a cross sectional area of $4.5t$. Substitution of this value into Eq. 6 provides a spring stiffness $k$:

$$k = \frac{18 \pi E_c t^2}{D}$$

In the finite element analyses, the value for $k$ was determined from Eq. 7 as well as values one half, twice, and four times that in Eq. 7. These four spring stiffnesses provided a good coverage between the sliding and hinged assumptions.

Figure 4 shows plots of the hoop tension versus distance from the bottom of a wall with a sliding base ($k=0$), a hinged base ($k=\infty$), and with the bottom of the wall supported radially by springs with stiffnesses specified earlier. The hoop tension is non-dimensionalized with respect to the product $wHR$, where $R$ is the tank radius. The tank geometry, $H^2/(D\ell)$, is 0.4. For the sliding base, the non-dimensionalized hoop tension varies linearly from 1.0 at the bottom to 0 at the top of the wall. For the hinged boundary, the hoop tension is 0 at the bottom increasing non-linearly to about 0.48 at the top. The hoop tensions for the walls supported at the bottom by a spring lie somewhere between these two extremes.

Figures 5 to 13 have similar curves for tank geometry values $H^2/(D\ell)$ of 0.8, 1.2, 1.6, 2.0, 4.0, 5.0, 6.0, and 8.0. As tanks become taller relative to their diameter, the effect of the boundary condition at the bottom of the wall on the upper part of the cylinder becomes less. For instance, in Fig. 13 the hoop tension in the upper half of the wall is virtually equal to that in a wall with sliding base.

The hoop tension curves in Figs. 4 to 13 will permit designers of cylindrical liquid manure tanks to determine the hoop force at any level in the wall, for any tank geometry now commonly used on farms. With this information and a set of design criteria appropriate for the design of a watertight tank, the wall thickness and the reinforcing steel can be determined (Jofriet et al. 1987).

The radial reaction at the bottom of the wall, $F_r$, causes meridional bending moments, $M(x)$, in the cylindrical shell.
They can be determined from (Roark and Young 1975):

\[ M(\lambda) = F_r \left( \frac{a_2 b_2}{a_1 \lambda} - \frac{a_3 b_3}{2a_1 \lambda} - \frac{b_1}{2\lambda} \right) \]  

(8)

in which:

\[ a_1 = \sinh^2(\lambda H) - \sin^2(\lambda H) \]
\[ a_2 = \cosh(\lambda H) \sinh(\lambda H) - \cos(\lambda H) \sin(\lambda H) \]
\[ a_3 = \sinh^2(\lambda H) + \sin^2(\lambda H) \]
\[ b_1 = \cosh(\lambda \tau) \sin(\lambda \tau) + \sinh(\lambda \tau) \cos(\lambda \tau) \]
\[ b_2 = \sinh(\lambda \tau) \sin(\lambda \tau) \]
\[ b_3 = \cosh(\lambda \tau) \sin(\lambda \tau) - \sinh(\lambda \tau) \cos(\lambda \tau) \]
\[ x = \text{distance from bottom of wall, and} \]
\[ \lambda = \left( \frac{3(1 - v^2)}{R^2 \ell^2} \right)^{\frac{1}{4}} \]

FOOTING DESIGN

The radial reaction, \( F_r \), at the bottom of the wall must be carried by the wall footing as a radial force. The radial force, \( F_r \), can be resisted by friction between the bottom of the footing and the soil, by passive soil pressure against the vertical side of the footing, by placing the footing against undisturbed ground, or by reinforcing the footing with hoop steel to resist the resulting hoop tension (Eq. 1), or a combination of these. The hoop force, \( T_f \), resulting from the radial force is simply the difference between the total hoop tension in the wall for the case being considered and that assuming a perfectly sliding base. In the latter case the force, \( F_r \), and hence the footing hoop tension, \( T_f \), is zero.

The total hoop force in a wall with geometry, \( H^2/(Dt) \), and spring support, \( k \), at the bottom can be found easily by integrating the hoop forces plotted in Figs. 4 to 13 over the height of the wall, i.e. by finding the area, \( \alpha \), under the curves in Figs. 4 to 13. Since the hoop force plots are non-dimensional, the areas so found have to be multiplied by \( wRH^2 \) thus providing a hoop force of \( \alpha wRH^2 \). Since the total hoop force

Table II: Values of \((0.5 - \alpha)\) for various wall geometries

<table>
<thead>
<tr>
<th>( H^2/(Dt) )</th>
<th>0</th>
<th>9</th>
<th>18</th>
<th>36</th>
<th>72</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.128</td>
<td>0.168</td>
<td>0.199</td>
<td>0.221</td>
<td>0.245</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>0.115</td>
<td>0.154</td>
<td>0.186</td>
<td>0.208</td>
<td>0.235</td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>0.100</td>
<td>0.138</td>
<td>0.170</td>
<td>0.192</td>
<td>0.221</td>
<td></td>
</tr>
<tr>
<td>1.6</td>
<td>0.088</td>
<td>0.124</td>
<td>0.154</td>
<td>0.176</td>
<td>0.205</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>0.087</td>
<td>0.120</td>
<td>0.148</td>
<td>0.169</td>
<td>0.195</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>0.072</td>
<td>0.100</td>
<td>0.124</td>
<td>0.141</td>
<td>0.163</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>0.067</td>
<td>0.091</td>
<td>0.111</td>
<td>0.124</td>
<td>0.142</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>0.064</td>
<td>0.086</td>
<td>0.103</td>
<td>0.114</td>
<td>0.129</td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>0.060</td>
<td>0.079</td>
<td>0.094</td>
<td>0.103</td>
<td>0.115</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td>0.055</td>
<td>0.071</td>
<td>0.082</td>
<td>0.089</td>
<td>0.099</td>
<td></td>
</tr>
</tbody>
</table>

in a wall with a sliding base is \( 0.5wRH^2 \), the hoop force transferred from the wall to the footing by shear is:

\[ T_f = (0.5 - \alpha) \omega \rho R H^2 \]  

(9)

Table II lists values of \((0.5 - \alpha)\) for all wall geometries and all spring stiffnesses provided in Figs. 4 to 13.

The hoop force in Eq. 9 assumes that the entire force, \( F_r \), is resisted by hoop tension (or compression if the load is radially inward). The reduction of this force by friction at the bottom of the footing is easy to estimate assuming a conservatively small coefficient of friction of say 0.4 to 0.5. The reduction from a passive soil pressure along the side of the footing is more difficult to estimate. It will be a function of the radial displacement and the type of soil. Of course, for any passive pressure to develop, the soil must be undisturbed and the footing concrete must be placed directly in contact with the undisturbed soil.

An upper estimate of the passive soil pressure, \( p_p \), may be obtained from:

\[ \lambda = \left( \frac{3(1 - v^2)}{R^2 \ell^2} \right)^{\frac{1}{4}} \]

Fig. 5. Hoop tension in the wall of tank with geometry ratio \( H^2/Dt = 0.8 \).

Fig. 6. Hoop tension in the wall of tank with geometry ratio \( H^2/Dt = 1.2 \).
where:

\[ p_p = K_p w_s H_s \]  \hspace{1cm} (10)

\[ K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \]  \hspace{1cm} (11)

where \( \phi \) = internal angle of friction. This angle of friction should be estimated conservatively low.

To develop the full passive pressure, there has to be considerable radial displacement of the footing. The precise relationship between radial displacement and passive soil pressure is very difficult to determine. This means that the design of hoop steel in the footing is extremely difficult and the conservative designer may well neglect the resistance from the soil completely.

To provide some guidance in this difficult task it may help to be able to estimate the radial displacement of the bottom of the tank wall given a known radial spring support. The radial displacement of the wall at any level is a linear function of the hoop strain, and therefore hoop stress, at that level providing the applied force is uniform or linearly varying. Thus the curves showing the variation of hoop force in Figs. 4 to 13 also depict the deformed shape of the wall, to some scale. Referring to Eq. 3 and replacing the radial force by \( 2T/D \) (see Eq. 1) and considering that for a unit height on the wall the area is \( 1t \), the radial displacement at any level can be written as:
The most important forces for which the wall of a cylindrical liquid manure tank must be designed were reviewed. The determination of the internal hoop forces in the wall of a cylindrical liquid manure tank is difficult if the boundary support at the bottom of the wall is other than sliding or hinged. Figures 4 to 13 provide an aid to designers who want to consider a more realistic boundary condition. This paper also provides the designer with sufficient information to calculate the horizontal shear force transferred from the wall to the footing given certain boundary conditions at the bottom of the wall. As well, the radial displacements can be determined. Friction and passive earth pressure can provide horizontal reactions at the wall footing; the frictional resistance is easy to calculate; the passive earth pressure can only be estimated as being less than the upper bound value. The net radial force equals the horizontal shear force minus the reactive forces; it must be resisted by the footing ring and will result in hoop tension to be resisted by reinforcing steel.

An example analysis of the hoop tension in a tank is in the Appendix.

ACKNOWLEDGEMENT

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REFERENCES


Example analysis

A liquid manure tank has to have a capacity of 500 m$^3$. A diameter of 16.0 m was selected. Thus a wall height $H = 2.50$ m provides the required storage. Estimate a wall thickness of 200 mm. The tank’s non-dimensional geometry ratio, $H^2/(D t)$, then is 1.95. Fig. 8 ($H^2/(D t) = 2.0$) can be used to provide hoop tension forces.

Assume a footing with 200 mm (= $t$) projections beyond the wall and a 300 mm (= $1.5 t$) thickness. This stiffer ring with a cross-sectional area, $A_f$, of 4.5$t^2$ can be modelled by a spring support at the bottom of the wall using Eq. 5.

$$K = 4\pi A_f E_c / D = 18\pi E_c t^2 / D$$

From Fig. 8 the hoop tensions from the liquid manure are:

<table>
<thead>
<tr>
<th>Height from bottom (m)</th>
<th>$T/wHR$</th>
<th>$T$ (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.375</td>
<td>75</td>
</tr>
<tr>
<td>0.25</td>
<td>0.425</td>
<td>85</td>
</tr>
<tr>
<td>0.50</td>
<td>0.455</td>
<td>91</td>
</tr>
<tr>
<td>0.75</td>
<td>0.470</td>
<td>94</td>
</tr>
<tr>
<td>1.00</td>
<td>0.465</td>
<td>93</td>
</tr>
<tr>
<td>1.25</td>
<td>0.430</td>
<td>86</td>
</tr>
<tr>
<td>1.50</td>
<td>0.400</td>
<td>80</td>
</tr>
<tr>
<td>1.75</td>
<td>0.350</td>
<td>70</td>
</tr>
<tr>
<td>2.00</td>
<td>0.280</td>
<td>56</td>
</tr>
<tr>
<td>2.25</td>
<td>0.215</td>
<td>43</td>
</tr>
<tr>
<td>2.50</td>
<td>0.145</td>
<td>29</td>
</tr>
</tbody>
</table>

The total tensile force, $T_{tot}$, obtained by integrating the tensile forces per unit of height in column 3 of the above table, is 187.5 kN; $\alpha$ in Eq. 8 is 0.375. The hoop tension in the footing, if other resistances to the radial base shear transferred to the footing are neglected, is:

$$T_f = (0.5 - \alpha) w R H^2 = (0.5 - 0.375) \times 10 \times 8 \times 2.5^2 = 62.5 kN$$

The radial displacement at the bottom of the wall, if $T_f$ is the only resisting force, is:

$$\Delta R = \frac{y w H R^2}{E_c t} = \frac{0.375 \times 10 \times 2.5 \times 8^2}{25,000,000 \times 0.2} = 0.00012 m = 0.12 mm$$

The radial shear force transferred from wall to footing to cause this radial displacement is:

$$F_r = T_f / R = 62.5 / 8.0 = 7.81 kN/m$$

If full passive soil resistance against the side of the footing were generated and if the angle of internal friction of the soil were estimated low, say $\Phi = 20^\circ$, and if the wall were backfilled to within 0.5 m of the top of the wall with soil weighing 20 kN/m$^3$ then by Eq. 10:

$$p_p = \frac{K_p H_s}{1 + \sin 20^\circ x 20 x 2.0} = 81.6 kN/m^2$$

Since the footing is 0.3 m thick, the passive resistance of the soil would be a maximum of 24.5 kN/m. A more conservative estimate of the soil resistance would be to assume an “at-rest” (zero strain) condition with a $K_o$ of about 0.5. In this case the resistance would be 20 kN/m$^2$, or 6 kN/m over the height of the footing.

As well, the friction resistance at the bottom of the footing can be considerable in this case. The wall and footing together weigh 16 kN/m. A conservative friction coefficient of 0.4 would provide a friction reaction with a maximum value of 6.4 kN/m.

With the information available, the designer has to make a judgment regarding the amount of hoop reinforcement in the footing. The choices are:

1. Reinforce for the full 62.5 kN hoop tension.
2. Reinforce for part of the 62.5 kN hoop tension.
3. Assume that the entire radial force of 7.81 kN/m is resisted by friction and soil resistance.

Any of these three approaches can be defended as being reasonable.

The maximum value of meridional bending moment by Eq. 7 is 2.33 kN·m per metre for a base reaction of 7.81 kN/m, occurring 0.7 m above the base. Assuming an uncracked section, this would cause a maximum bending stress of 0.35 MPa tension on the outside face, compression on the inside.