A micromechanics model for predicting dynamic loads during discharge in bulk solids storage structures

Q. Zhang and M.G. Britton

Department of Biosystems Engineering, University of Manitoba, Winnipeg, Manitoba, Canada R3T 5V6.

Zhang, Q. and Britton, M.G. 2003. *A micromechanics model for predicting dynamic loads during discharge in bulk solids storage structures*. Canadian Biosystems Engineering/Le génie des biosystèmes au Canada 45: 5.21 - 5.27. A two-dimensional microstructure mechanics model has been developed for predicting dynamic pressures during discharge in bulk solids storage bins. The model assumes that the bulk solids consist of elastic discs of equal size and dynamic lateral pressures on the bin walls are induced by dilation of the stored bulk solids. Dilation during discharge is calculated from the change in the microscopic structure of bulk solids. A micromechanics theory is used to transfer microscopic dilation to macroscopic stresses, or dynamic pressures, on the bin walls. Predicted dynamic pressures are in close agreement (within 8%) with the reported experimental data for a smooth walled and a corrugated model bin with an H/D (height/diameter) ratio of 1.5 filled with barley. The model predictions also compare favorably with the recommendation of ASAE EP 433 for typical full-size bins for storing wheat.

Un modèle constitutif en deux dimensions pour la prédiction des pressions dynamiques se développant à l’intérieur de matériaux solides granulaires à l’échelle de la microstructure lors de la vidange de silos d’entreposage a été développé. Dans ce modèle, les matériaux granulaires sont représentés par des disques élastiques de taille uniforme et la dilatation de ces matériaux est la cause des pressions latérales dynamiques exercées sur les parois des silos. La dilatation durant la vidange est déterminée à partir des variations microscopiques dans la structure des matériaux granulaires. Un modèle constitutif à l’échelle de la microstructure des matériaux granulaires est utilisé afin de calculer les contraintes internes causées par cette dilatation et pour convertir ces contraintes en pressions latérales sur les parois des silos. L’écart entre les valeurs de pression latérale prédites par le modèle et des données expérimentales recueillies sur un silo doté de parois lisses ainsi que de parois en tôle ondulée et présentant un rapport hauteur/diamètre égal à 1,5 a été inférieur à 8%. Les valeurs prédites par le modèle sont également en accord avec les recommandations de la norme ASAE EP 433 dans le cas de silos pleine grandeur utilisés pour l’entreposage du blé.

INTRODUCTION

Dynamic loads during discharge (emptying) are the most frequent causes of structural failures of bulk solids storage facilities. Among the proposed hypotheses on the mechanism by which dynamic loads are induced, the following three have been accepted to some extent: (1) pressure switch (Jenike and Johanson 1968); (2) impact effect (Kmita 1991); and (3) dilation (shear-induced volume increase) of bulk solids (Smith and Lohnes 1980). In comparing with liquid storage structures in which no dynamic loads develop during discharge, it becomes apparent that the frictional behaviour of bulk solids is the key to understanding dynamic loads. Of the above mentioned three hypotheses, the dilation hypothesis is founded on a fundamental frictional characteristic of bulk solids − dilatancy.

It is a well known fact that particulate materials generally dilate when subjected to shearing because of interlocking between particles (Das 1983). When bulk solids are discharged from storage bins shearing occurs in the mass of stored materials causing bulk solids to dilate. However, dilation is restricted by bin walls, and consequently loads on bin walls increase. Various dilatancy theories have been developed in the area of soil mechanics. Based on these theories, several models have been proposed for predicting dynamic loads in bulk solids storage structures (Xu et al. 1993; Zhang et al. 1994). These models have been derived from macroscopic approaches. A fundamental assumption is that bulk solids behave like continua, i.e. the discontinuous nature of bulk solids is not directly incorporated in these predictive models.

When external stresses are imposed on a bulk material, they are carried by the contacts between particles. Macroscopic deformations of the medium result from interparticle movements (slip and rotation) and particle deformations. This fundamental microscopic characteristic of bulk solids cannot be directly considered in continuum theories. The objective of this study was to develop a model from the perspective of micromechanics theory to predict discharge loads in bulk solids storage structures.

**REVIEW OF MICROSCOPIC THEORY AND MODEL FOR STATIC LOADS**

Xu et al. (1996) developed a microscopic predictive model for static loads in bulk solids storage structures. The model is based on a microstructural theory proposed by Granik and Ferrari (1993) for idealized granular media which consist of elastic particles (discs in 2D) of equal size, packed hexagonally (Fig. 1a). A doublet (a pair of particles) is considered as a basic structural unit in a granular medium. In a hexagonal structure, an individual particle is involved in six doublets which are separated by structural angle $\gamma = 60^\circ$. When the granular medium is subjected to loads macroscopically, strains develop within the doublet because of deformations, rotation, and slipping of particles. These strains are termed microstrains. Corresponding to microstrains, microstresses also develop...
within the doublet. The three microstress components are $p_1$, $p_2$, and $p_3$, which are stresses uniformly distributed over the projected area of the particle, in the directions of the lines connecting the centres of the contacting particles (Fig. 1b). Microstresses are related to macrostresses through Eqs. 1, 2, and 3 (Granik and Ferrari 1993):

\begin{align*}
\sigma_x &= (p_1 + p_2) \cos^2 \gamma + p_3 \\
\sigma_y &= (p_1 + p_2) \sin^2 \gamma \\
\tau_{xy} &= (p_1 - p_2) \cos \gamma \sin \gamma
\end{align*}

where:
- $\sigma_x$ = macroscopic stress in x-direction,
- $\sigma_y$ = macroscopic stress in y-direction,
- $\tau_{xy}$ = macroscopic shear stress,
- $p_\alpha$ = microstresses, $\alpha = 1, 2, 3$, and
- $\gamma$ = micro-structural angle ($60^\circ$).

Kinematic relationships can be used to relate microstrains to macrostrains:

\begin{align*}
\varepsilon_1 &= \varepsilon_x \cos^2 \gamma + 2 \varepsilon_y \sin \gamma \cos \varepsilon_x \sin^2 \gamma \\
\varepsilon_2 &= \varepsilon_x \cos^2 \gamma - 2 \varepsilon_y \sin \gamma \cos \varepsilon_y \sin^2 \gamma \\
\varepsilon_3 &= \varepsilon_x
\end{align*}

where:
- $\varepsilon_x$ = macroscopic strain in x-direction,
- $\varepsilon_y$ = macroscopic strain in y-direction,
- $\varepsilon_{xy}$ = macroscopic shear strain, and
- $\varepsilon_\alpha$ = microstrains, $\alpha = 1, 2, 3$.

For linear elastic particles, microstresses are related to microstrains through the microscopic modulus of elasticity (Granik and Ferrari 1993):

$$p_\alpha = \sum_{\beta=1}^{3} A_{\alpha\beta} \varepsilon_\alpha = \sum_{\beta=1}^{3} E \delta_{\alpha\beta} \varepsilon_\alpha$$

where:
- $A_{\alpha\beta}$ = microscopic modulus of elasticity,
- $\delta_{\alpha\beta}$ = Kronecker delta, and
- $E$ = macroscopic modulus of elasticity.

Based on the microstructural theory outlined in Eqs. 1 to 7, Xu et al. (1996) developed a model for predicting static pressures in bulk solids storage structures:

\begin{align*}
\sigma_{0x} &= \left(\rho g + \lambda \tan^2 \gamma \right) \frac{R}{\mu} \left[1 - \exp \left(-\frac{\mu y}{R \tan^2 \gamma}\right)\right] \\
\sigma_{0y} &= (\sigma_{0x} - \lambda y) \tan \gamma \\
\lambda &= \left(\frac{\rho g}{2 \sin^2 \gamma}\right) \left(v \sin^2 \gamma - \cos^2 \gamma\right)
\end{align*}

where:
- $\sigma_{0x}$ = static stress in x-direction,
- $\sigma_{0y}$ = static stress in y-direction,
- $\rho$ = bulk density,
- $g$ = acceleration due to gravity,
- $\mu$ = coefficient of static friction between bulk solids and structure,
- $v$ = Poisson’s ratio of particles,
- $R$ = hydraulic radius of bin,
- $y$ = material depth, and
- $\lambda$ = intermediate variable.

Equations 8 - 10 have been shown to be adequate in predicting static loads in grain bins (Xu et al. 1996). In this study, a model has been developed for use with the model of Xu et al. (1996) to predict dynamic loads during discharge in bulk solids storage structures.

**MODEL DEVELOPMENT FOR DYNAMIC LOADS**

When a bulk solids material is discharged from a storage structure, the material flows towards the discharge outlet at different velocities within the structure, e.g. with the highest velocity at the bin center and lowest near the walls, if the outlet is centrally located. Velocity differences cause shearing to occur within the bulk. When a particulate material is sheared, a macroscopic shear plane develops in the direction of the externally applied shear force $\tau$ (Fig. 2a). The actual motion of particles, however, occurs along a wavy plane because particles have to roll over each other (Nemat-Nasser 1980). In a typical direct shear box, the wavy plane divides the bulk into two portions, with each behaving like a rigid block (Horne 1965;
Shear stress, $\tau$

Dilatancy angle, $\psi$

Normal stress, $\sigma_n$

Macroscopic shear

Microscopic shear

Dilation

Fig. 2. Shear surfaces in granular medium under direct shear condition: (a) direct shear box; (b) rigid block model.

Nemat-Nasser 1980). This characteristic of shearing is modeled as sliding between two serrated faces of rigid blocks (Fig. 2b) (Rowe 1962). As one block moves over the other one vertically, it is also forced by the serrated surfaces to move laterally against the normal force $\sigma_n$, thus the volume of the bulk increases. This shear-induced volume increase is termed dilatancy. If the material is stored in a bin, the bin walls prevent lateral expansion of the material. Therefore, lateral pressure increases, i.e. “dynamic” pressure occurs. The above discussion also implies that the discharge pressure is not truly dynamic. The discharge pressure results from the changes in the internal structure of the bulk solids and is not directly associated with the kinetic energy. Therefore, the predictive model derived in the following sections is based on the static equilibrium.

For a hexagonal structure consisting of equal sized particles, the maximum dilation on an individual shear surface (bands) is calculated as (Fig. 3):

$$\omega = d \left(1 - \sin \gamma \right)$$

where:

$\omega = $ lateral dilation, and

$d = $ diameter of particles.

When a granular material is discharged from a bin, individual particles move in quasi-rigid blocks and a number of instantaneous shear surfaces (bands) may form in the bin (Rong et al. 1995). Assuming that the average width of these instantaneous blocks is $b$, then the number of shear surfaces across the bin is $(D/b - 1)$, and the total amount of dilation across the bin is $(D/b - 1)\omega$, where $D$ is the bin diameter. If the grain mass is allowed to deform freely, the (maximum) strain caused by dilation in the lateral direction would be:

$$\varepsilon_{dx} = \frac{\left( \frac{D}{b} - 1 \right) \omega}{D}$$

Fig. 3. Dilation of hexagonal structure caused by vertical shearing.

for $6$, $7$, and $13$, the horizontal microstress caused by dilation is determined as:

$$p_{d3} = E \varepsilon_{d3} = E \frac{1}{\xi} \left(1 - \sin \gamma \right)$$

where: $p_{d3} = $ dynamic microscopic stress in the x-direction.

It should be noted that $p_{d3}$ is a net increase in horizontal microstress during discharge. In other words, the total (dynamic) microstress in the horizontal direction is $p_3 = p_{d3} + p_{d3}$, where $p_{d3}$ is the static horizontal microstress. Similarly, the net dynamic microstresses in the other two directions are $p_{d1}$ and $p_{d2}$, respectively. To evaluate $p_{d1}$ and $p_{d2}$, the equilibrium of dynamic macrostresses on a differential element is examined in the vertical direction (Fig. 4):

$$\frac{\partial \sigma_{dy}}{\partial x} + \frac{\partial \sigma_{dy}}{\partial y} - f = 0$$

where:
The net dynamic lateral pressure on the bin wall is now calculated by combining Eqs. 14, 19, and 17:

\[
\sigma_{dy} = E \left( \frac{1}{\xi} - \frac{d}{D} \right) (1 - \sin \gamma) \exp \left( \frac{\mu_d y}{R \tan^2 \gamma} \right) \quad (20)
\]

The total dynamic pressure, defined as the sum of static and net dynamic pressures, is determined by summing Eqs. 8 and 20.

**DISCUSSION**

Comparing Eq. 20 with the methods that are recommended in most design codes and standards for predicting discharge loads indicates that the model parameters which may not be readily obtained are the macroscopic modulus \(E\) and the coefficient of shear block thickness \(\xi\). An empirical equation proposed by Smith and Lohnes (1983) may be used for determining \(E\) for several agricultural grains:

\[
E = 3(1 - 2v) \left( \frac{\sigma_h}{K_i \varepsilon_{v,ult}} + 1 \right)^2 K_i \quad (21)
\]

where:
- \(\sigma_h\) = hydrostatic stress,
- \(\varepsilon_{v,ult}\) = asymptotic value of volumetric strain, and
- \(K_i\) = initial macroscopic bulk modulus.

Insufficient information exists in the literature for determining the thickness of shear blocks (or the number of shear bands). Therefore, the coefficient of shear block thickness \(\xi\) was estimated by fitting Eq. 20 to experimental data reported by Zhang et al. (1993) for two model bins filled with wheat. The two bins, one smooth walled and one corrugated, were both 1.0 m in diameter and 1.5 m high. Static lateral pressures measured at 1.12 m from the free surface were 3.3 and 2.6 kPa for smooth walled and corrugated bins, respectively, and dynamic pressures were 5.2 and 3.8 kPa for the two bins, respectively. Material parameters used in load predictions are summarized in Table 1. The values for the initial macroscopic bulk modulus and the asymptotic volumetric strain were reported by Smith and Lohnes (1983) for wheat. The bulk density, internal friction angle, and wall friction coefficients were given by Zhang et al. (1993). The static friction coefficients are used to approximate the dynamic friction coefficients in load predictions. A Poisson’s ratio of 0.29 is assumed (Xu et al. 1996) and the average equivalent diameter of wheat kernels is considered to be 4 mm (Mohsenin 1986). Substituting these parameters into Eqs. 8 and 10, static lateral pressures are predicted to be 2.9 and 2.5 kPa for the smooth walled and corrugated bins, respectively. These values are in reasonable (13 and 6%, respectively) agreement with the corresponding measured static pressures.

Letting the dynamic pressure be equal to the measured value and solving Eq. 20 for \(\xi\), two \(\xi\)-values of 60 and 105 are obtained for the smooth and corrugated bins, respectively. It should be mentioned that the thickness of shear blocks changes instantaneously during discharge. The above estimated values were the “nominal”

---

**Table 1. Material parameters used in model predictions for wheat and barley.**

<table>
<thead>
<tr>
<th>Material parameter</th>
<th>Wheat</th>
<th>Barley</th>
<th>ASAE*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk density, (\rho) (kg/m(^3))</td>
<td>779</td>
<td>638</td>
<td>834</td>
</tr>
<tr>
<td>Initial macroscopic bulk modulus, (K_i) (kPa)</td>
<td>645</td>
<td>141</td>
<td>645</td>
</tr>
<tr>
<td>Asymptotic value of volumetric strain, (\varepsilon_{v,ult})</td>
<td>7.3%</td>
<td>14.0%</td>
<td>7.3%</td>
</tr>
<tr>
<td>Equivalent diameter of kernel, (d) (mm)</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Poisson’s ratio of kernel, (v)</td>
<td>0.29</td>
<td>0.40</td>
<td>0.29</td>
</tr>
<tr>
<td>Micro-structural angle, (\gamma)</td>
<td>60°</td>
<td>60°</td>
<td>60°</td>
</tr>
<tr>
<td>Angle of internal friction, (\phi)</td>
<td>25.6°</td>
<td>23.0°</td>
<td>22.0°</td>
</tr>
<tr>
<td>Coefficient of friction on corrugated wall, (\mu)</td>
<td>0.43</td>
<td>0.40</td>
<td>0.37</td>
</tr>
<tr>
<td>Coefficient of friction on smooth wall, (\mu)</td>
<td>0.22</td>
<td>0.18</td>
<td>-</td>
</tr>
</tbody>
</table>

* Parameter values from ASAE EP 433 (2003) for wheat in corrugated bins
Fig. 5. Variation of predicted overpressure factor with coefficient of shear block thickness for a 1.5-m high and 1.0-m diameter model bin filled with wheat.

Fig. 6. Comparison of overpressures between model prediction and ASAE EP 433 (2003) recommendation for a 10-m high and 5-m diameter corrugated steel bin filled with wheat.

Predicted dynamic pressures are compared with experimental data reported by Zhang et al. (1993) for barley in the two model bins described earlier (one smooth walled and one corrugated). The material parameters in Table 1 for barley are used in model predictions. The values for the initial macroscopic bulk modulus, the asymptotic volumetric strain, and Poisson’s ratio were from Xu et al. (1997). The bulk density, internal friction angle, and wall friction coefficients were reported by Zhang et al. (1993). Predicted pressures are in close agreement with the experimental data for both $\xi = 60$ and 105 (Table 2).

Table 2. Comparisons of model predictions with data of Zhang et al. (1993) for barley in two model bins 1.5 m high and 1.0 m diameter.

<table>
<thead>
<tr>
<th></th>
<th>Static pressure (kPa)</th>
<th>Dynamic pressure (kPa)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Predicted</td>
<td>Difference</td>
</tr>
<tr>
<td>Smooth walled</td>
<td>2.8</td>
<td>2.8</td>
<td>0%</td>
</tr>
<tr>
<td>Corrugated</td>
<td>2.2</td>
<td>2.8</td>
<td>-6%</td>
</tr>
</tbody>
</table>
et al. (1993) for barley in two model bins, one smooth walled and one corrugated. Model predictions of dynamic pressures are in close agreement with that of ASAE EP 433 (2003) for wheat in a typical full-size corrugated bin with a height to diameter ratio (H/D) of 2.5. Higher overpressure factors are predicted for greater H/D ratios.

AKNOWLEDGEMENT

The authors are grateful to the Natural Science and Engineering Research Council of Canada for financially supporting the project.

REFERENCES


NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>cross-sectional area of bin</td>
</tr>
<tr>
<td>b</td>
<td>average width of shear blocks</td>
</tr>
<tr>
<td>A_{eg}</td>
<td>microscopic modulus of elasticity</td>
</tr>
<tr>
<td>d</td>
<td>diameter of particles</td>
</tr>
<tr>
<td>D</td>
<td>bin diameter</td>
</tr>
<tr>
<td>E</td>
<td>macroscopic modulus of elasticity</td>
</tr>
<tr>
<td>f</td>
<td>dynamic friction force per volume of differential element</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity</td>
</tr>
<tr>
<td>H</td>
<td>bin height</td>
</tr>
<tr>
<td>K_i</td>
<td>initial macroscopic bulk modulus</td>
</tr>
<tr>
<td>p_{\alpha}</td>
<td>microstresses, \alpha 1, 2, 3</td>
</tr>
<tr>
<td>p_{dy}, p_{dz}</td>
<td>dynamic microscopic stresses in y and z directions, respectively</td>
</tr>
<tr>
<td>p_{dz}</td>
<td>dynamic microscopic stress in the x-direction</td>
</tr>
<tr>
<td>p_{dz}</td>
<td>static horizontal microstress</td>
</tr>
</tbody>
</table>
\( p_{13} \)  static horizontal microstress
\( R \)  hydraulic radius of bin
\( S \)  circumference of bin
\( y \)  material depth
\( \alpha \)  running index, 1, 2, 3
\( \beta \)  running index, 1, 2, 3
\( \delta_{\alpha\beta} \)  Kronecker delta
\( \Delta y \)  thickness of differential element
\( \varepsilon_{\alpha} \)  microstrains, \( \alpha \) 1, 2, 3
\( \varepsilon_{dx} \)  lateral strain caused by dilation
\( \varepsilon_{\text{v,ult}} \)  asymptotic value of volumetric strain
\( \varepsilon_{x} \)  macroscopic strain in \( x \)-direction
\( \varepsilon_{xy} \)  macroscopic shear strain
\( \varepsilon_{y} \)  macroscopic strain in \( y \)-direction
\( \gamma \)  micro-structural angle (60°)
\( \lambda \)  intermediate variable
\( \mu \)  coefficient of static friction between bulk solids and structure
\( \mu_d \)  coefficient of dynamic friction of wall
\( \nu \)  Poisson’s ratio of particles
\( \xi \)  coefficient of shear block thickness
\( \rho \)  bulk density
\( \sigma_{dx} \)  static stress in \( x \)-direction
\( \sigma_{dy} \)  static stress in \( y \)-direction
\( \sigma_{dx} \)  net dynamic macrostress in \( x \)-direction
\( \sigma_{dy} \)  net dynamic macrostress in \( y \)-direction
\( \sigma_h \)  hydrostatic stress
\( \sigma_n \)  normal stress
\( \sigma_x \)  macroscopic stress in \( x \)-direction
\( \sigma_y \)  macroscopic stress in \( y \)-direction
\( \tau \)  shear stress
\( \tau_{dx,y} \)  net dynamic macroscopic shear stress
\( \tau_{xy} \)  macroscopic shear stress
\( \Psi \)  dilatancy angle
\( \omega \)  lateral dilation