INTRODUCTION

Discharge records published by such agencies as the Water Resources Branch of the Department of Northern Affairs are usually in the form of average daily discharges. Such records are of little value for studying the peak instantaneous flows, particularly on small watersheds. The obvious method of obtaining the peak instantaneous flows on a watershed is by review of the recorded stage hydrographs. Generally, however, these records are not readily available. Further, the task of analysis is extremely time consuming if a long period of record is involved. An alternative method to the above which is much more expedient, is to utilize the average daily discharges by applying a conversion factor to these flows to derive the peak instantaneous discharges.

CURRENT PROCEDURES

Two approaches are reported in the literature for resolving the relation between the peak instantaneous discharge with another flow parameter and/or a geomorphic property of the watershed. Fuller (3) in 1914 studied the relationship between the peak instantaneous discharge and the average 24-hr flows of floods on large basins located in the Eastern United States. He developed a relationship between these variables of the form,

\[ Q_{\text{max}} = QA^{-0.30} \]

where \( Q_{\text{max}} \) = maximum recorded instantaneous discharge (cfs),
\( Q \) = largest average rate of flow over a 24 consecutive hour period which was recorded during the same period of years as \( Q_{\text{max}} \) (the two discharges did not necessarily occur during the same flood), and
\( A \) = drainage basin area in sq. miles.

In establishing the "excess ratio", \((Q_{\text{max}} - Q)/Q\), of equation 1, Fuller rationalized that for all floods, \( Q_{\text{max}} \) would be greater than \( Q \). In addition, it was postulated that the "excess ratio" would decrease with an increase in drainage basin size. For larger watersheds, the rate of runoff is generally high for at least 24 hrs because the runoff producing storm is of considerable duration whereas on small basins a cloudburst may cause a flood which will give runoff over a few hours resulting in a large \( Q_{\text{max}} \) and only a moderate \( Q \).

Langbein (5) employed a different approach to solve the problem by using only flow records. He established a relationship between the peak instantaneous discharge and maximum daily discharge in terms of (a) the ratio of the daily discharge of the preceding day to the discharge of the maximum day, and (b) the ratio of the daily discharge of the succeeding day to that of the maximum day. A plot of these ratios as presented by Langbein is given in figure 1.

INVESTIGATIONS

Because of the success and general acceptance of the methods given by Fuller and Langbein in resolving the complex interrelationship between the peak instantaneous flows and the average daily flows for the streams they studied, it was assumed similar methods could be successfully employed to establish the properties for Prairie Streams. To accomplish this objective, discharge hydrographs were collected from 22 small watersheds, which varied in size from 12 - 1000 square miles, located within the central region of Canada. All watersheds selected were equipped with automatic water level recorders which had been in operation at least five years. In total, about 260 hydrographs, including both rainfall and snowmelt runoff events were studied.

Details of the watersheds as to their station description, location and size are given in table 1 and figure 2 respectively. As shown on the figure, the watersheds are widely separated geographically, and hence they represent a wide range in physiographic conditions.
In an attempt to minimize the effect of regional influences in the investigations, the watersheds were separated to the regional groups listed in Table 1.

### TABLE I. DESCRIPTION OF HYDROMETRIC STATIONS

<table>
<thead>
<tr>
<th>Group</th>
<th>Code</th>
<th>Name</th>
<th>Location</th>
<th>WRB No.</th>
<th>Area $^a_{Ag}$</th>
<th>Area $^a_{Ae}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foothills</td>
<td>1.1</td>
<td>North Branch</td>
<td>near Twin Butte, Alta.</td>
<td>SAD 16</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>North Fork Milk</td>
<td>above U.S. St.</td>
<td>11AA 03</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>1.3</td>
<td>Stinson Ck.</td>
<td>near Pekisko.</td>
<td>5BL 3</td>
<td>96</td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>1.4</td>
<td>Highwood R.</td>
<td>near Diebel's</td>
<td>5BL 19</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Cypress Hills</td>
<td>2.1</td>
<td>Lyons Coulee</td>
<td>at International boundary</td>
<td>11AB 25</td>
<td>66</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>Woodpile Coulee</td>
<td>near International boundary</td>
<td>11AB 01</td>
<td>68</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>2.3</td>
<td>Battle Ck.</td>
<td>at Ranger Station, Sask.</td>
<td>11AB 31</td>
<td>78</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>2.4</td>
<td>East Fork Battle Ck.</td>
<td>near International boundary</td>
<td>11AB 03</td>
<td>111</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>Manyberries</td>
<td>at Brodin's Farm</td>
<td>5AF 12</td>
<td>127</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>2.6</td>
<td>Sage Ck.</td>
<td>at Q Ranch near Wildhorse, Alta.</td>
<td>11AA 20</td>
<td>174</td>
<td>149</td>
</tr>
<tr>
<td>Wood Mountains</td>
<td>2a.1</td>
<td>Horse Ck.</td>
<td>near International boundary</td>
<td>11AE 03</td>
<td>74</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>2a.2</td>
<td>East Branch</td>
<td>at International boundary</td>
<td>11AE 1</td>
<td>534</td>
<td>190</td>
</tr>
<tr>
<td>Central</td>
<td>3.1</td>
<td>Little Boggy Ck.</td>
<td>at Cote, Sask.</td>
<td>5MD 6</td>
<td>60</td>
<td>15b</td>
</tr>
<tr>
<td></td>
<td>3.2</td>
<td>Indianhead Ck.</td>
<td>near Indian Head, Sask.</td>
<td>5JL 2</td>
<td>203</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>3.3</td>
<td>Boggy Ck.</td>
<td>above Junction</td>
<td>5JF 6</td>
<td>175</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>3.4</td>
<td>Qu'Appelle R.</td>
<td>above Buffalo Pound L.</td>
<td>5JG 6</td>
<td>932</td>
<td>222</td>
</tr>
<tr>
<td>Manitoba Escarpment</td>
<td>4.1</td>
<td>Scott Ck.</td>
<td>near Laurier, Man.</td>
<td>5LJ 26</td>
<td>26</td>
<td>15b</td>
</tr>
<tr>
<td></td>
<td>4.2</td>
<td>McKinnon Ck.</td>
<td>near McCreary, Man.</td>
<td>5LJ 27</td>
<td>30</td>
<td>17b</td>
</tr>
<tr>
<td></td>
<td>4.3</td>
<td>Garland R.</td>
<td>near Garland, Man.</td>
<td>5LG 2</td>
<td>30</td>
<td>22b</td>
</tr>
<tr>
<td></td>
<td>4.4</td>
<td>Edwards Ck.</td>
<td>near Dauphin, Man.</td>
<td>5LJ 30</td>
<td>67</td>
<td>38b</td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>Mink R.</td>
<td>near Ethelbert, Pine River, Man.</td>
<td>5LJ 19</td>
<td>82</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>4.6</td>
<td>Pine R.</td>
<td>near</td>
<td>5LG 1</td>
<td>84</td>
<td>48b</td>
</tr>
</tbody>
</table>

$^a_{Ag}$ is the gross drainage area in square miles and $^a_{Ae}$ the effective area in square miles.

$^b$ Indicates an estimated effective area.

### Peak Ratio - Area Relationships

Prairie hydrologists have long recognized the error of using the gross drainage area in rational flood flow formulae when applied to Prairie streams characterized by large amounts of depressional storage. Under these conditions, the area of the watershed which contributes to flow during any given runoff event varies with the magnitude of the flood as different channels and depressions become connected to the main stream system.

Perhaps, the most striking feature of the arrays was the large scatter of the
data which represents a wide range in values for the peak ratio with different storms. In addition, several other features were exhibited by the plots.

(a) For watersheds within a selected regional grouping there was a trend for the peak ratio to decrease as the size of the basin increased. This characteristic is to be expected as large watersheds have high base flows. In addition, the hydrograph from large watersheds have long time bases so that q_p is more nearly equal to q_2, and the ratio closer to unity.

(b) For watersheds within a given regional grouping, the peak ratios from rainfall and snowmelt events were different. In general, it was found that for the watersheds in the Foothills group, the peak ratios of snowmelt events tended to be higher than those from rainfall. Conversely, an opposite trend was apparent in the arrays for watersheds in the Cypress Hills and Wood Mountain group (figure 3) and those in the Central Plains. Unfortunately, there were not sufficient data available either on watersheds within the same regional grouping or for watersheds in different physiographic regions to permit a complete study of this phenomena.

Average peak ratios of summer storm events and effective area

As a further reduction to the data, the average values of the peak ratios of summer storm events were determined and plotted with effective area as shown in figure 4. As indicated on this figure, the average peak ratios from watersheds within a given physiographic region can be related to effective area in equational form as,

\[ \frac{q_p}{q_2} = CA_e^{-n} \]

where \( A_e \) = effective area in sq. miles, and \( C, n \) = coefficient and exponent.

Values of the coefficient, \( C \), and exponent, \( n \), for the different regional groups are tabulated in table 2.

### TABLE II. PEAK RATIO—EFFECTIVE AREA RELATIONSHIPS OF WATERSHEDS IN DIFFERENT REGIONAL GROUPS

<table>
<thead>
<tr>
<th>Group</th>
<th>Equation</th>
<th>Range of Validity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{q_p}{q_2} = 3.9 A_e^{-0.22} )</td>
<td>60 - 300 mi²</td>
</tr>
<tr>
<td>2 and 2a</td>
<td>( \frac{q_p}{q_2} = 19 A_e^{-0.46} )</td>
<td>50 - 200 mi²</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{q_p}{q_2} = 3.7 A_e^{-0.36} )</td>
<td>45 - 225 mi²</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{q_p}{q_2} = 11 A_e^{-0.30} )</td>
<td>15 - 50 mi²</td>
</tr>
</tbody>
</table>

### Discussion

The coefficient, \( C \), of equation 2 can be considered as an index of the flashiness of the streams. As shown in table 2, those watersheds which fall in Cypress Hills and Wood Mountain (Groups 2 and 2a) are very flashy in comparison with the others. Streams of this region are of the intermittent type carrying little or no base flow during the summer months. In addition, the area is covered by relatively-sparse, vegetal cover, the soil mantle is thin and the relief very pronounced. In the region, runoff-producing storms during the summer months are primarily short, intense, thunderstorms. The combination of physiographic and climatic elements of the region tend to produce hydrographs with a short period of rise and rapid recession. In contrast, watersheds in the Foothills although on steep topography do not tend to be nearly as flashy. This can be attributed to the fact that, (a) most of the basins are situated in the lee of the Rockies and do not receive intense cloudbursts but are affected primarily by long-duration cyclonic-type storms, and (b) the watersheds are densely forested thus the streams are of the permanent type with high base flows. On watersheds in the Central Plains, because of their poor relief and drainage development, storage masks the flashiness caused by the intense storms and attenuates the peak flows.

To indicate the interrelationship between the width of the hydrograph and the peak ratios, the average ratio, \( q_p/q_2 \) for each watershed was plotted against the average time in hours during which the stage was above 75 percent of the peak stage as measured above base flow, \( T_{75} \) (see figure 5). As shown in the figure, there is a distinct trend for \( q_p/q_2 \) to decrease as the width of the hydrograph increases. Further, when \( T_{75} \) is greater than approximately 24 hrs the ratio tends toward a limit of unity.

The exponent of area, \( n \), in equation 2, is an index of the relative importance of area as opposed to other factors in contributing to the peak ratio. The streams in Groups 2 and 2a and 3 have the largest exponents being -0.46 and -0.36 respectively whereas the exponents of the equation for Groups 1 and 4 are smaller. Gray (4) has shown that although area is only one of the many factors of the geomorphic regime that establishes hydrograph shape, it is often predominant inasmuch as within a given physiographic region it reflects many other watershed factors (lengths and slopes of streams) which affect the time distribution of surface runoff.

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Figure 4. Regional trends in peak ratio—Area relation.

Figure 5. Average peak ratio and the average duration.

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3. The potato will dissipate the applied stress when held at a constant strain. For periods up to ten minutes the relaxation curve may be adequately represented by a three-term exponential equation. For periods greater than ten minutes it would be necessary to use a four-term equation.

4. Although testing the material in compression with small diameter loading plungers leads to a complex stress distribution, a reasonable approximation of the modulus of elasticity may be obtained by considering the potato sample as an infinite elastic solid being loaded under a perfectly rigid loading plunger. The relaxation curve may be adequate for periods up to ten minutes. For periods greater than ten minutes it would be necessary to use a four-term equation.

5. The potato shows an increase in the modulus of elasticity with an increase in the rate of compression strain.

REFERENCES


ACKNOWLEDGMENTS

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The Authors—G. E. Timbers, L. M. Staley and E. L. Watson are respectively, former graduate research assistant (now Research Officer, Engineering Research Service, Canada Department of Agriculture) and Associate Professors of Agricultural Engineering, University of British Columbia.

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SUMMARY

Discharge records collected from 22 watersheds located within the central region of Canada were analyzed to establish relationships between the peak instantaneous discharge, $q_p$, and the average daily discharge, $q_d$, for Prairie streams. From analysis of the data it was found that the variation in the average peak ratio, $q_p/q_d$, of summer storm events for watersheds within selected regional groupings could be related to the effective size of the drainage basin. Differences in the relationships for watersheds in different physiographic areas could be attributed to such factors as topography, climate and vegetation as they affect the discharge characteristics of streams. In addition, it was found that the peak ratios from rainfall and snowmelt events on watersheds within a given regional grouping were not of the same magnitude and could not be considered from a homogenous population. This necessitates that a separate relationship be established for the different events.

A graph is presented which can be used to estimate the peak instantaneous discharge for Prairie streams using published average daily flows for; (a) the day preceding the occurrence of the peak rate, $q_1$, (b) the average daily flow on the day following the peak, $q_3$, and (c) the day following the occurrence of the peak rate.

LITERATURE CITED

tion obtained by neglecting it sustains the ignored qualification. By corollary, a solution can be obtained by ignoring the second qualification and checking the validity of the solution by determining whether or not the ignored qualification is sustained by the solution.

Ignoring the second power qualification and substituting for \( Y_3 \) in the functional, the problem appears as

\[
Z = 300Y_1 + 100Y_2
\]

\[
\frac{200}{Y_1} + \frac{200}{Y_2} = 100
\]

These equations solve readily by Lagrange’s method to yield:

\[
Y_1 = \frac{\sqrt{300} - \sqrt{100}}{2} \times 2 = 3.15 \text{ acres per hour}
\]

\[
Y_2 = \frac{\sqrt{100} + \sqrt{300}}{2} \times 2 = 5.47 \text{ acres per hour}
\]

\[
Y_3 = 63.0 \text{ hp}
\]

from which \( t_1 = 63.5 \) hours and \( t_2 = 36.5 \) hours

and where, incidentally \( Z = \$1,492.00 \)

The validity of the solution is proven by the second qualification being sustained by substitution.

\[
Y_3 = 63.0 > 54.7
\]

The combination of Lagrange’s method and linear programming provides a powerful approach to problems involving both variable-cost and fixed-cost relationships. A sequence can be devised as follows:

1. Solve a linear program without the inclusion of machine size entries to obtain acreages, or guess at final acreages.
2. Using these acreages, solve the minimum cost machinery combination using Lagrange’s method.
3. Using time values \( t_i \) obtained from equation 2, enter the machine-size relationships in the program matrix in the manner illustrated in figure 1.
4. Solve the program and compare the final acreages with the acreages obtained from equation 1.

These four steps constitute one iteration of a combined “solve and verify” program. If there is lack of agreement, a second iteration can be performed starting at step 2 and using acreages obtained from equation 4.

CONCLUSIONS

A programming solution obtained in this manner insures the achievement of a final program in which machinery is sized and the scale of enterprises determined simultaneously, with full interaction between the fixed - cost of machinery and the returns from the enterprises.

This procedure, although at first sight appearing tedious, has been employed frequently by the author without difficulty.

Results of these applications have provided substantiative data on machinery requirements. Observation of these data constitute another paper. In general, such observations do not support the contention that machinery fixed - costs are the principle factor in a cost-price squeeze.

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... RUNOFF FREQUENCY

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be used to describe the population except for one year in which an unusual rainstorm of 3.74 inches resulted in a peak estimated at 960 cfs. For the fourteen years of record, the peak flows originated from snowmelt except for three years when peaks resulted from sudden rainstorms. An analysis of the peak flows originating from rain alone is not warranted because of insufficient data.

Peak flows calculated by the Forsaith equation\(^1\) (3) superimposed on the measured values for the Davin watershed (figure 3), approach the measured values in the low flow range, but tend to deviate as the peak flows increase. The calculated values of Forsaith were all higher than the measured values for different probabilities. This may be explained by the fact that the data used by Forsaith were obtained from areas larger than the Davin watershed in which factors affecting watershed yield may not have been as predominant. In addition, the use of a V-notch weir for measuring flow is considered to be more accurate than stream-gauging techniques.

\(^1\)Forsaith equation is:

\[ Q = C(32.3 \ A^{0.5} \ T^{0.44}) \]

where: \( Q \) is the peak flood in cfs that may be anticipated to be equalled or exceeded, on an average, once in a period of \( T \) years; \( C \) is the runoff coefficient, the value of which depends on the watershed characteristics and geographic location of the drainage area; \( A \) is the watershed area in square miles, and \( T \) is the frequency period in years.

SUMMARY AND CONCLUSIONS

Watershed studies conducted indicate that a wide range of runoff volumes and peak flows can be expected from a 2987-acre watershed. Ninety percent or more of the total runoff resulted from snowfall which accounted for approximately a third of the annual precipitation. The measured peak flows from 1951 to 1964 were found to be less than that estimated from the Forsaith equation. Results to date indicate that more than 14 years of data are required to establish a reliable frequency analysis. Extreme events have occurred and further knowledge of these events would be of value for design purposes.

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