Optimizing the Design of Solar Energy Greenhouses

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Abstract
A solar energy greenhouse technology developed in northern China for winter vegetable production has shown a good potential for Manitoba winter conditions. The design and performance of solar energy greenhouses are dependent on the geographical location. Theoretical analysis was conducted to optimize the design of solar greenhouses with respect to the latitude of the location for maximum solar radiation gain. The parameters that were analyzed included the greenhouse length and the roof slope. The model predictions indicate that shading of the solar storage (north) wall in a solar energy greenhouse may considerably reduce the amount of solar energy stored in the greenhouse. The tilt angle of the north roof can be optimized to eliminate its shading effect. To reduce the shading effect by the end wall, a long greenhouse is preferred.

Keywords: solar energy, greenhouse, shading
INTRODUCTION

In cold climates, cost of heating is the major expense in operating greenhouses in the winter season. In fact, high heating costs usually prohibit winter greenhouse production in winter months in most part of Canada. Alternative energy sources, such as solar energy and biomass energy, may provide an economically feasible solution. In a previous study (Beshada et al. 2005), a solar energy greenhouse (SEG) was shown the promise of using solar energy to meet the greenhouse heating requirement during winter in Manitoba. The main components of this solar energy greenhouse were a passive heat storage system and thermal insulation of the solar window at nighttime. The passive storage system in the greenhouse is a solid (sand fill) wall, which forms part of the greenhouse envelope on the north side, or the north wall. For maximum energy storage, the north wall should be exposed to all sources of radiation during daytime. In the current design, both the north roof and end walls shade the north wall to a certain degree. The objective of this paper is to develop a theoretical solar energy model to predict shading on the north wall for optimizing the design of solar greenhouses for maximum solar gain.

METHODOLOGY

Description of Solar Energy Greenhouses

The solar energy greenhouse (SEG) consists of steel framing, a plastic cover, a solar energy collection (north) wall, and a thermal blanket (fig. 1). The greenhouse is constructed so that during the daytime the inside surface of the north wall is exposed to direct solar radiation. The north wall, which is filled with sand, acts as heat reservoir and it also blocks the northern wind, reducing heat losses caused by infiltration. As the air in the greenhouse cools down at nighttime, solar energy absorbed by the wall would be radiated back to the room. For the maximum collection of solar energy, the greenhouse was oriented in the east-west direction. The thickness of the north wall is about 30 cm, of which the portion filled with sand is 15 cm and insulation 15 cm. The 15-cm fibreglass insulation on the north and sidewalls provided thermal resistance of about R-20. While the north wall and a small section of insulated roof form the enclosure on the north side of the greenhouse, the plastic cover forms the greenhouse enclosure on the south side, which is the solar window of the greenhouse. The plastic cover is a single layer of 6-mil polyethylene. To minimize heat loss through the plastic cover at night, a thermal blanket is placed over the plastic cover. A winch system is used to operate the thermal blanket, i.e., rolling it up at daytime and placing it over the plastic cover at nighttime.
Energy of Solar Radiation

Generally there are three sources of radiation that reach the north wall, the direct/beam $I_b$, sky diffuse $I_{ds}$, and albedo/ground reflected $I_{dg}$. The total incident solar radiation $I$ on a horizontal surface is the sum of the beam and sky diffuse and given as:

$$I = I_b + I_{ds}$$  \hspace{1cm} (1)

The total solar energy on a tilted surface is related to the total solar radiation on a horizontal surface by a tilt factor $f$ as follows (Duffie and Beckman, 1980):

$$I_t = I f = I_b f_b + I_{ds} f_{ds} + I_{dg} f_{dg}$$  \hspace{1cm} (2)

where:
- $I_t = \text{total solar radiation on the tilted surface, W/m}^2$  
- $I = \text{total solar radiation on a horizontal surface, W/m}^2$  
- $I_b = \text{beam radiation, W/m}^2$  
- $I_{ds} = \text{sky diffused radiation, W/m}^2$  
- $I_{dg} = \text{ground reflected diffuse solar radiation, W/m}^2$  
- $f = \text{overall tilt factor}$  
- $f_b = \text{tilt factor for beam radiation}$

Figure 1. The side view of solar energy greenhouse.
The albedo is a function of the ground reflectivity $\rho$, which is 0.7 for fresh snow cover and 0.2 without snow (Liu and Jordan, 1977), as expressed in equations 3.

$$I_{ds} = \rho(I_b + I_{ds})$$  \hspace{1cm} (3)

The daily diffuse radiation on a horizontal surface is correlated to the clearness of the sky, or the clearness index. Since the monthly average daily total radiation is recommended for design purposes (Duffie and Beckman, 1980; Hsieh, 1986), the monthly average daily diffuse radiation is calculated using the monthly average clearness index $k$ as follows (Liu and Jordan, 1977).

$$\frac{I_{ds}}{I} = 1.390 - 4.027k + 5.531k^2 - 3.108k^3$$  \hspace{1cm} (5)

The “bar” above the variable indicates the monthly average value for the variable. The monthly average clearness index is the ratio of monthly average daily total solar energy on a horizontal surface and the monthly average daily extraterrestrial radiation $\bar{I}_o$ on a horizontal surface.

$$k = \frac{\bar{I}}{\bar{I}_o}$$  \hspace{1cm} (6)

Depending on the geographic latitude, declination and sunset hour angle, the daily extraterrestrial radiation in Wh/m$^2$ is related to the solar constant and the day of year as follows.

$$I_o = 7.64I_{cs}(1 + 0.033\cos\left(\frac{360n}{365}\right))(\frac{2\pi s}{360}\sin\beta\sin\delta + \cos\beta\cos\delta\sin\sigma)$$  \hspace{1cm} (7)

where:

$I_o$ = daily extraterrestrial radiation in Wh/m$^2$
$I_{cs}$ = solar constant, 1353 W/m$^2$
$\beta$ = geographic latitude
$\delta$ = declination
$\omega_s$ = sunset hour angle
$n$ = day of the year

The monthly average of the daily extraterrestrial radiation can be calculated using the same equation (equation 7) by using $n$ and $\delta$ for the mean day of the month (Duffie and Beckman, 1980). According to Hsieh (1986), the declination angle $\delta$ and hour $\omega$ angles are calculated as follows:

$$\delta = 23.45\sin\left[280.11 + 0.986n\right]$$  \hspace{1cm} (8)
\[ \omega = \pm \frac{1}{4} m \]  

(9)

Where \( m \) is the number of minutes from the local solar noon. The hour angle at sunset and sunrise is also calculated using the latitude of the location and declination.

\[ \cos \omega_s = -\tan \beta \tan \delta \]  

(10)

The tilt factors \( f_{ds} \) and \( f_{dg} \) for diffuse radiation are fractions of the slope of the surface \( \gamma \) and have a view factor to the sky and ground, \( \cos \gamma \), while the beam radiation tilt factor \( f_b \) is a function of latitude \( \beta \), surface inclination to the horizontal \( \gamma \), declination angle, and hour angle. Mathematically they are expressed as follows (Liu and Jordan, 1977; Duffie and Beckman, 1980; Hsieh, 1986).

\[
\begin{align*}
  f_{ds} &= \frac{1 + \cos \gamma}{2} \\
  f_{dg} &= \frac{1 - \cos \gamma}{2} \\
  f_b &= \frac{\cos(\beta - \gamma) \cos \delta \sin \sigma_s' + (\pi/180)\sigma_s' \sin(\beta - \gamma) \sin \delta}{\cos \beta \cos \delta \cos \sigma_s + (\pi/180)\sigma_s \sin \beta \sin \delta}
\end{align*}
\]  

(11)

(12)

Where \( \omega_s' \) is the sunset hour angle for the tilted surface and calculated as the minimum of the two values given below.

\[
\sigma_s' = \min \left[ \cos^{-1}(-\tan \beta \tan \delta), \cos^{-1}(-\tan(\beta - \gamma)\tan \delta) \right]
\]  

(13)

The surface tilt angle for the north wall is 90°, therefore, equation 11 gives the value of the diffuse radiation tilt factor to be 0.5. Assuming the winter period with total snow cover, the ground reflectivity is 0.7 (Liu and Jordan, 1977). Substituting equations, 11 and 12 into 3 and rearranging those using equations 1 and 4, the monthly average solar radiation on the north wall is determined as a function of geographic latitude, declination and hour angles as follows.

\[
\bar{I}_s = (\bar{I} - \bar{I}_{ds}) \left( \frac{\sin \beta \cos \delta \sin \sigma_s' - (\pi/180)\sigma_s' \cos \beta \sin \delta}{\cos \beta \cos \delta \sin \sigma_s + (\pi/180)\sigma_s \sin \beta \sin \delta} \right) + 0.35\bar{I} + 0.50\bar{I}_{ds}
\]  

(14)

**Shading of North Roof**

The maximum monthly average daily solar radiation to reach on the solar wall is given by equation 14. However, this value is reduced by the shaded fraction of the wall. In this section the effect of north roof will be optimized for minimum or no shading effect during winter on the heat storage wall. In addition, the maximum summer shading effect can also be considered as an asset to reduce ventilation load. The possible design of the
north roof shown in figure 2 (also fig. 1) is used to find the fraction of shaded north wall
\( f_{rs} \) as a function of geographical location and solar position.

Figure 2. The solar position and north wall shading by the north roof.

The fraction of north wall \( f_{rs} \) shaded due to the overhang of north roof is related to
the vertical shading angle \( \theta_1 \) and can be calculated using the following equation (Sharp,
1982).

\[
f_{rs} = \frac{x_1 \tan \theta_1 - h_4}{h_2}
\]  

(15)

The vertical shading angle can also be expressed in terms of the solar altitude angle \( \alpha \) and
incident angle \( i \) of the radiation as follows,

\[
\tan \theta_1 = \frac{\sin \alpha}{\cos i}
\]  

(16)

Both solar altitude and incident angles are functions of geographic latitude, declination
angle, and hour angle of the location. They are given as follows for a south facing vertical
surface (Duffie and Beckman, 1980).

\[
\sin \alpha = \sin \beta \sin \delta + \cos \beta \cos \delta \cos \omega
\]  

(17)

\[
\cos i = \sin \beta \cos \delta \cos \omega - \cos \beta \sin \delta
\]  

(18)
Substituting equations 17 and 18 into 16 yields:

$$ \tan \theta_1 = \frac{\sin \beta \sin \delta + \cos \beta \cos \delta \cos \omega}{\sin \beta \cos \delta \cos \omega - \cos \beta \sin \delta} $$

(19)

Since the highest solar altitude is at noon, the solar position at this time should be used to optimize the slope of the north roof. Using the solar position at noon results in a 0° hour angle and equation 19 can be reduced to:

$$ \tan \theta_1 = \frac{1}{\tan(\beta - \delta)} $$

(20)

Inserting equation 20 into 15 relates the fraction of north wall shading at noon to the geographic latitude, declination of the day, the height of the north wall and the rise of the north roof (h4-h3), and the horizontal span (x1) as follows:

$$ f_{rs} = \frac{x_1}{h_2 \tan(\beta - \delta)} - \frac{h_1}{h_2} $$

(21)

However, if the north roof is designed so that no shading can occur on the north wall, then h1 and h2 must be equal. To achieve zero shading, frs in equation 21 is forced to be zero, which results in equation 22.

$$ \tan \psi = \frac{h_4 - h_3}{x_1} = \frac{1}{\tan(\beta - \delta)} - \frac{h_3}{x_1} $$

(22)

Where \( \psi \) is the slope of the north roof for zero shading.

To find out the average daily shading effect \( \bar{f}_{rs} \), the equations 15 and 19 are combined and integrated over the sunset \( \omega_s \) and sunrise \( \omega_r \) hour angles as follows.

$$ \bar{f}_{rs} = \left( \frac{x_1}{h_2} \int_{\omega_r}^{\omega_s} \frac{\sin \beta \sin \delta + \cos \beta \cos \delta \cos \omega}{\sin \beta \cos \delta \cos \omega - \cos \beta \sin \delta} \, d\omega \right) - \frac{h_1}{h_2} $$

(23)

The reduction in the daily total solar radiation \( I_{ts} \) due to the north roof shading can be approximated using equations 14 and 23:

$$ \bar{I}_{ts} = \bar{f}_{rs} I_t $$

(24)

**Shading of End Walls**

The end walls usually shade the north wall in the morning and in the afternoon. The amount of shading due to the end walls depends both on the solar altitude and solar azimuth. The shading diagram of this condition is depicted in figure 3. The vertexes A
and B show the position where the solar ray intersects the north wall and a side wall, respectively.

![Figure 3. The solar position and north wall shading due to side walls.](image)

Assuming the shade on the north wall to be ellipse with radii $x_3$ and $h_2$, the amount of shading due to an end wall is given as a ratio of the shaded area and the whole area of the north wall as follows.

$$f_{ss} = \frac{\left(\frac{x_3 + h_7}{2}\right)^2 \times \pi}{4wh_2} = \frac{\pi\left(x_3 + h_2\right)^2}{16wh_2} \quad (25)$$

Assuming the side wall is a quarter of a circle and $h_7$ to be half of the wall height ($h_2/2$), using trigonometric relation from figure 3, the shaded fraction is related to the profile shade angle $\theta_2$ as follows:

$$f_{ss} = \frac{\pi}{64wh_2} \left(\frac{h_2 + 2h_2 \tan \theta_2}{\tan \theta_2}\right)^2 \quad (26)$$

Using the trigonometric relations in figure 3, the profile shading angle $\theta_2$ can also be related to the solar altitude and azimuth by equation 27.

$$\tan \theta_2 = \frac{\tan \alpha}{\sin \phi} \quad (27)$$
The following equation relates the solar azimuth to the solar altitude, declination and hour angles (Duffie and Beckman, 1980).

\[
\sin \phi = \cos \delta \frac{\sin \sigma}{\cos \alpha}
\]  

(28)

Using equations 17 and 27, equation 26 can be rewritten to relate the shading profile angle to the local parameters of latitude, declination and hour angles.

\[
\tan \theta_2 = \frac{\sin \beta \sin \delta + \cos \beta \cos \delta \cos \omega}{\cos \delta \sin \sigma}
\]  

(29)

Equations 26 and 29 can be used to calculate the shaded fraction of the north wall due to the side wall as a function of geographic parameters and the height and the width of the side wall.

The shaded fraction \( f_s \) due to the overhang north roof and the side wall is determined as follows.

\[
\tilde{f}_s = \tilde{f}_{rs} + \tilde{f}_{ss}
\]  

(30)

Once the shading fraction is determined, the total daily effective amount of solar energy \( I_{\text{eff}} \) that can be received on a north wall of width \( w \) and height \( h_2 \) can be estimated using equation 31 as follows.

\[
I_{\text{eff}} = h_2 w \tilde{I}_1 \left(1 - \tilde{f}_s\right)
\]  

(31)

**Shading of Knee Wall**

The knee wall is the vertical wall sometimes constructed along the perimeter of the south facing section. Figure 4 shows a typical knee wall and its dimensions.

Considering the greenhouse length \( l \) and the corresponding knee wall height \( h_k \) without shading the north wall can be calculated based on the solar position and direction of incident radiation. Using triangles AOC and BOC, the following trigonometric relation can be obtained.

\[
\frac{BC}{AO} = \frac{\sin \alpha}{\cos i}
\]  

(32)

Since BC and AO are the values representing the height of knee wall \( h_k \) and length of greenhouse \( l \), equation 32 can be rewritten as
\[ h_k = l \frac{\sin \alpha}{\cos i} \] 

(33)

By substituting equations 17 and 18 into 33, the maximum knee wall height can be calculated as a function of greenhouse length, geographic latitude and solar declination angle of the location.

\[ h_k = l \frac{\sin \beta \sin \delta + \cos \beta \cos \delta \cos \omega}{\sin \beta \cos \delta \cos \omega - \cos \beta \sin \delta} \] 

(34)

Figure 4. The solar position and the shading effect of the knee wall.

RESULTS AND DISCUSSION

The shaded fraction of the north wall due to the north roof varies with the time of the year. Figure 5 shows the variations in shading of north wall for three locations: 43°N (London, ON), 49°N (Winnipeg, MB), and 58°N (Churchill, MB) for a greenhouse described by Beshada et al. (2005). For example, no shading occurs in the 49°N location in December and January, and most part of February. If the greenhouse is used in March, the tilt of the north roof needs to be changed to avoid shading.
Figure 5. The north wall fraction shaded at various latitudes during different months of the year.

The optimal (no shading) tilt angle for the north roof is shown in figure 6. The angle of the north roof used in the current design is about 34°. For example, at latitude of 49°N, to obtain no shading until end of April the minimum angle of the north roof should be 55°. However, this tilt angle should be increased to 60° for 43°N and decreased to 46° for latitude of 58°N.

Figure 6. The minimum tilt angle of north roof required for zero shading condition at various locations and different months.

The shading effect of the end wall for various times of the year is shown in figure 7 for 49°N. The shading fraction was calculated for the time between 8:00 and 16:00.
hours for a north wall 100 ft long. As shown in the figure the shading in December could be as high as 35%.

![Graph showing shaded fraction over time](image)

**Figure 7.** The fraction of north roof shaded due to the side wall in different months for southern Manitoba (49°N).

**CONCLUSIONS**

Shading of the solar storage (north) wall in a solar energy greenhouse may considerably reduce the amount of solar radiation reaching the wall. Shading by the north roof decreases with the latitude of the location. Properly choosing the tilt angle of the north roof can eliminate the shading effect. Shading by the end wall is significant in winter months. The greenhouse should be as long as possible to reduce the relative area of shading caused by the end wall.

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**References**


